Friction Problems

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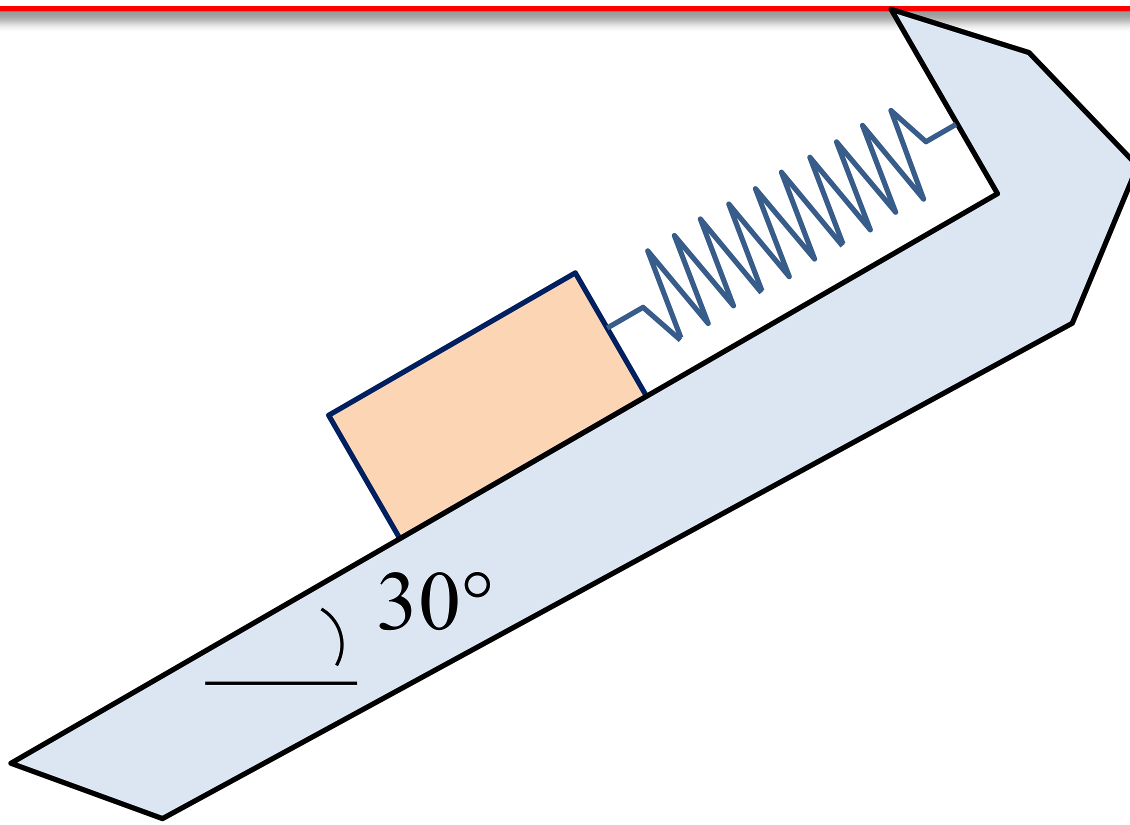
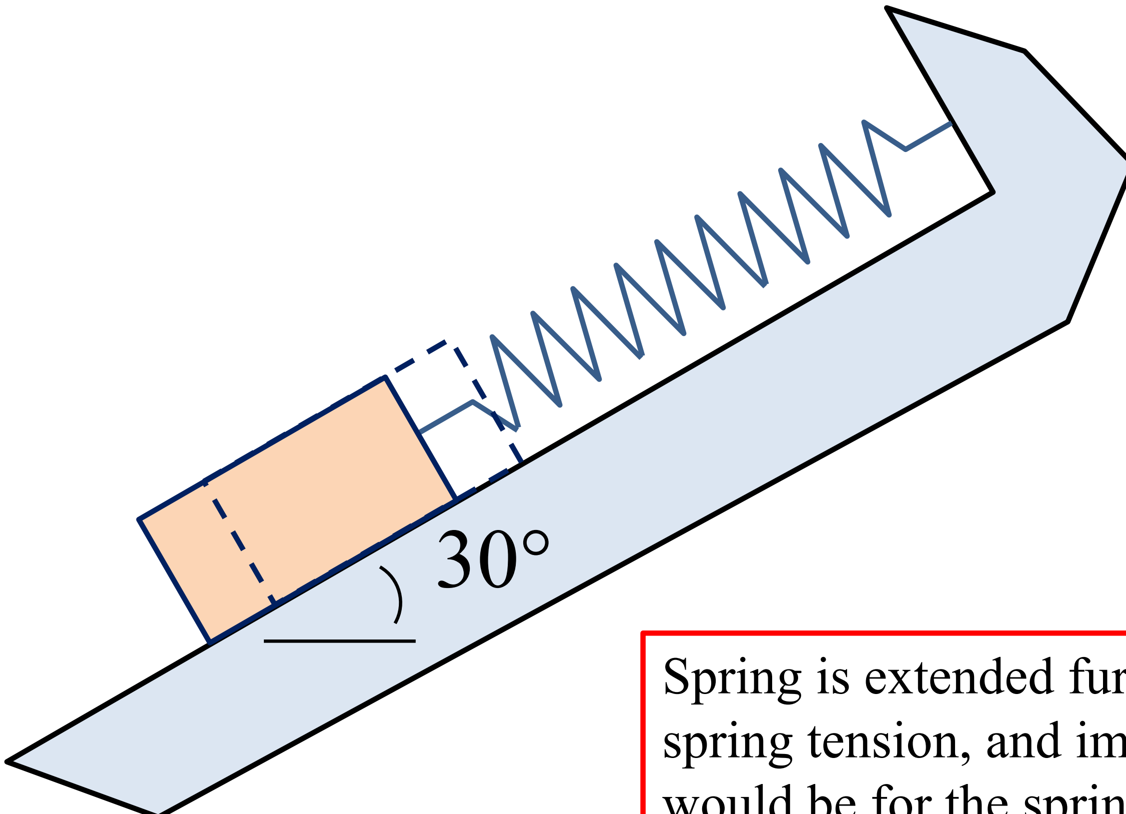
A 100 lb crate rests on an inclined surface and is attached to a spring as shown. If the coefficient of static friction between the crate and the surface is 0.35 find:

1. The maximum tension in the spring for which the crate will not slip when released;
2. The minimum tension in the spring for which the crate will not slip when released;
3. Will the block be in equilibrium when the spring tension is 60 lb? If so, calculate the friction force.

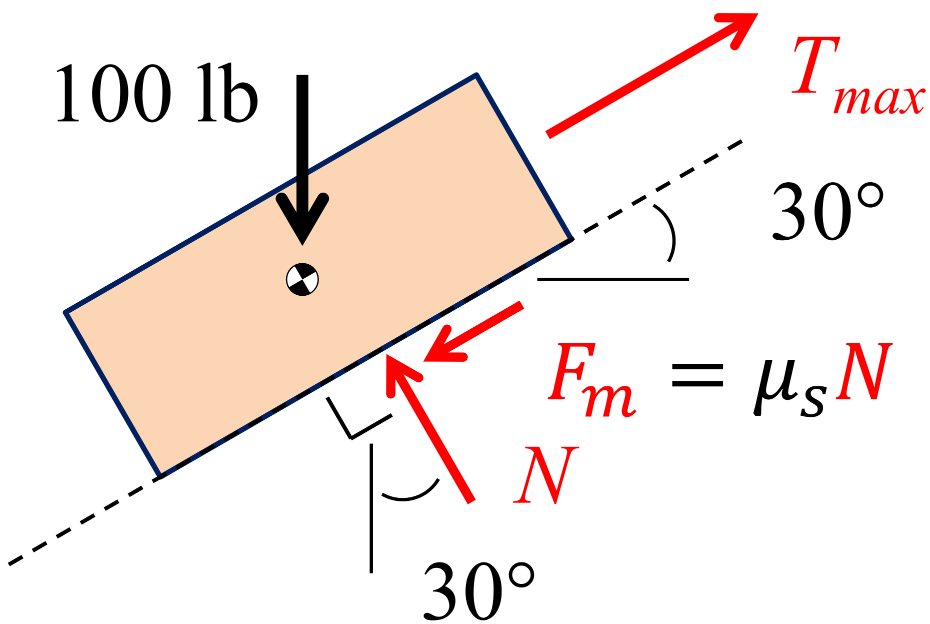


Condition for Maximum Spring Tension



Spring is extended further, increasing spring tension, and impending motion would be for the spring to pull the crate up the incline.

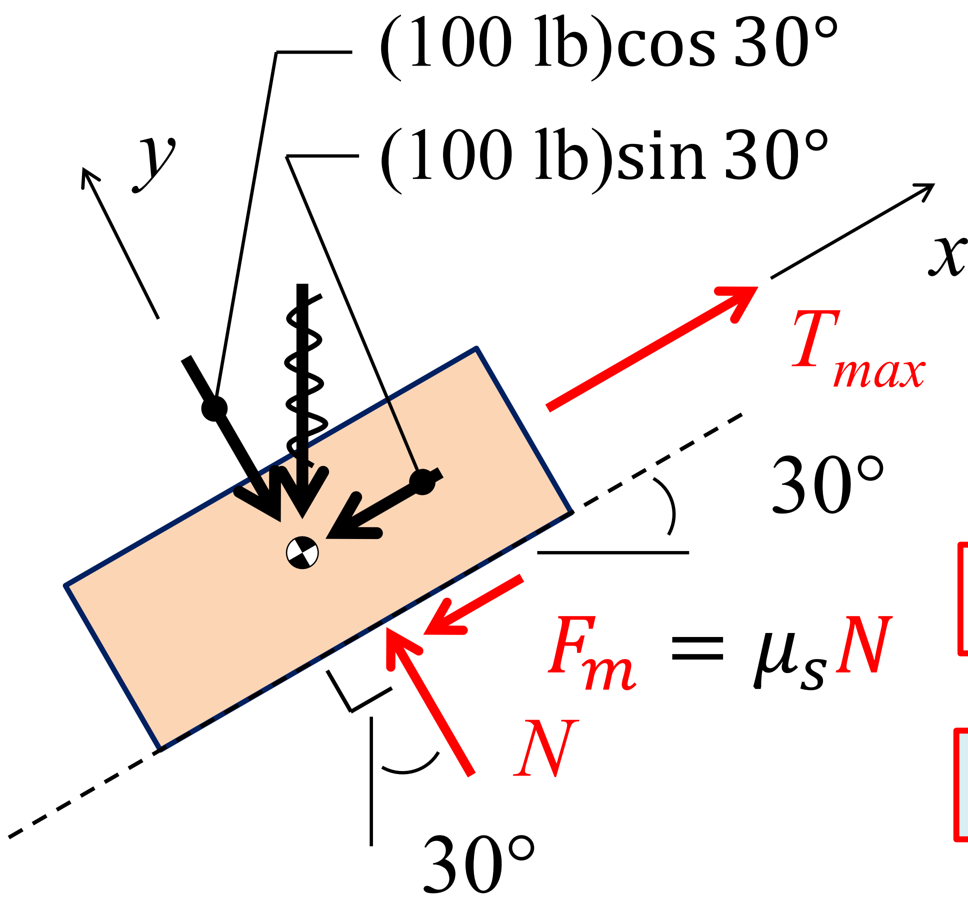
Free-Body Diagram of Block at Impending Motion (Maximum Spring Tension)



Notes

- Friction force is opposite impending motion;
- Three forces on block – must be concurrent (i.e. two equations of equilibrium available to solve for unknown forces;
- Convenient to orient reference coordinate system inclined with the surface;
- Two unknowns (N, T_{max}).

Free-Body Diagram of Block at Impending Motion (Maximum Spring Tension)

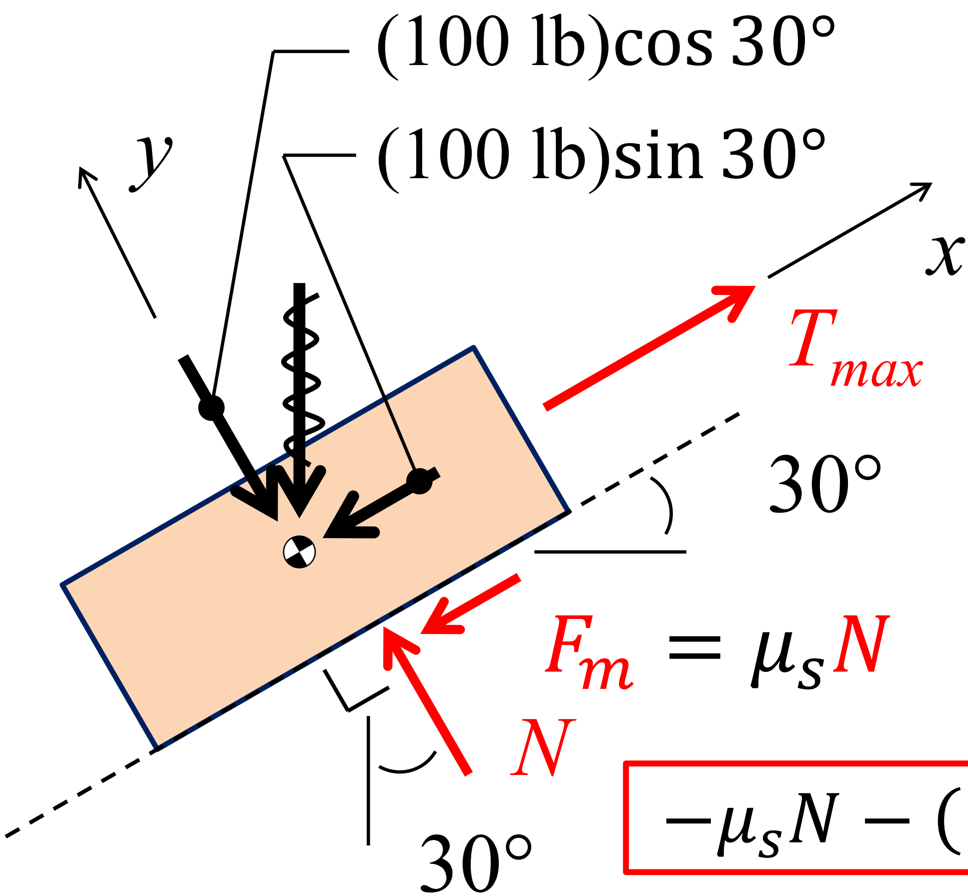


$$+\uparrow \sum F_y = 0$$

$$N - (100\text{ lb})\cos 30° = 0$$

$$N = (100\text{ lb})\cos 30°$$

Free-Body Diagram of Block at Impending Motion (Maximum Spring Tension)



$$\xrightarrow{+} \sum F_x = 0$$

$$-\mu_s N - (100\text{ lb})\sin 30° + T_{max} = 0$$

$$T_{max} = \mu_s N + (100\text{ lb})\sin 30°$$

Solve Equations of Equilibrium

1\. $N = (100 \text{ lb}) \cos 30°$

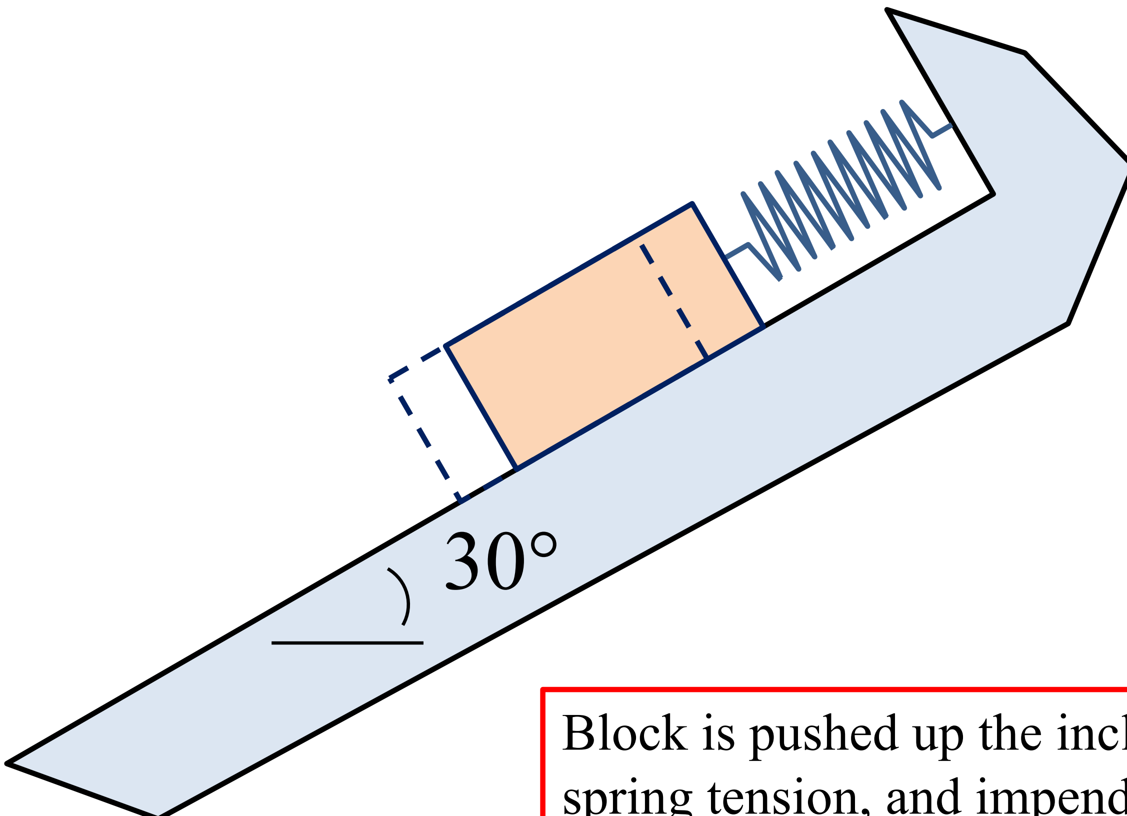
2\. $T_{max} = \mu_s N + (100 \text{ lb}) \sin 30°$

$T_{max} = \mu_s (100 \text{ lb}) \cos 30° + (100 \text{ lb}) \sin 30°$

$\mu_s = 0.35$

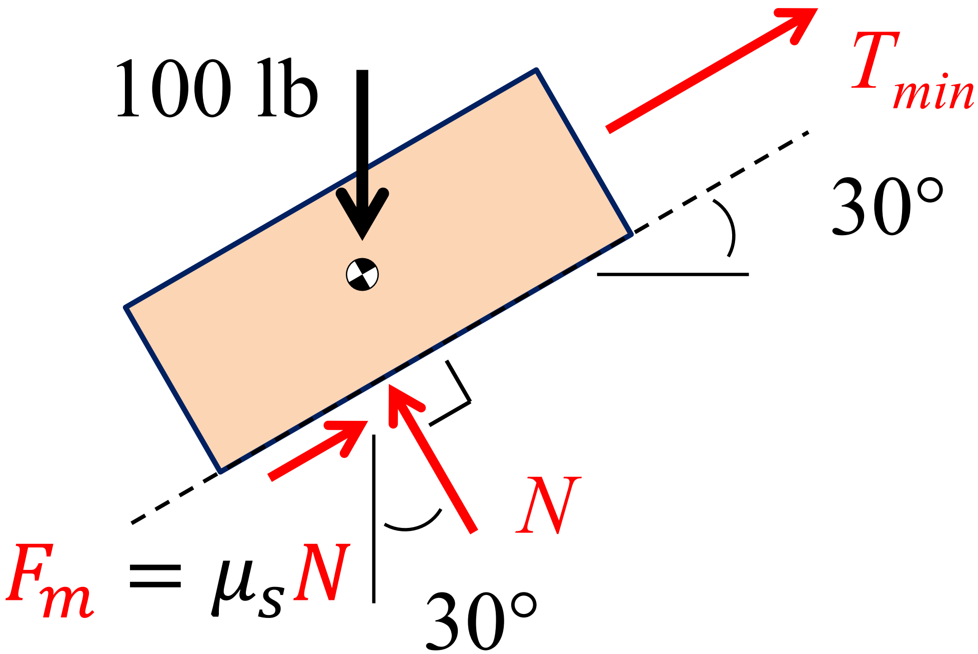
$T_{max} = 80.3 \text{ lb}$

Condition for Minimum Spring Tension



Block is pushed up the incline, decreasing spring tension, and impending motion would be for the weight of the crate to move the crate down the incline.

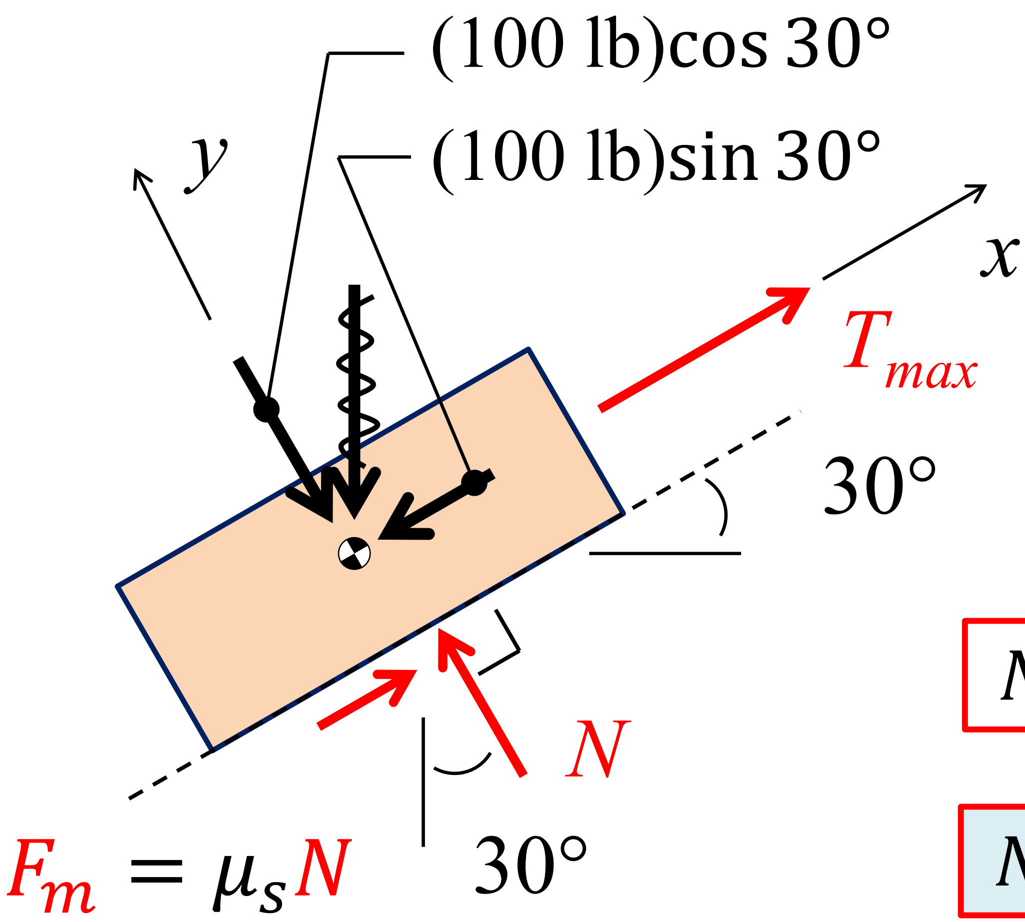
Free-Body Diagram of Block at Impending Motion (Minimum Spring Tension)



Notes

- Friction force is opposite impending motion;
- Three forces on block – must be concurrent (i.e. two equations of equilibrium available to solve for unknown forces;
- Convenient to orient reference coordinate system inclined with the surface;
- Two unknowns (N, T_{max}).

Free-Body Diagram of Block at Impending Motion (Minimum Spring Tension)

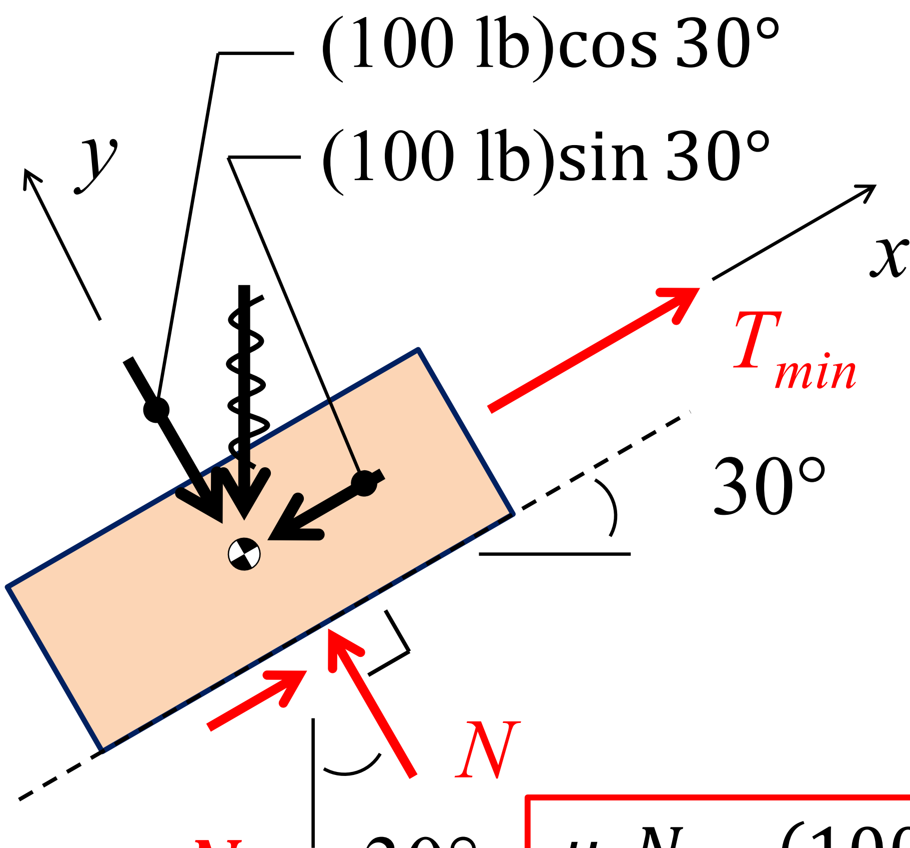


$$+\uparrow \sum F_y = 0$$

$$N - (100\text{ lb})\cos 30° = 0$$

$$N = (100\text{ lb})\cos 30°$$

Free-Body Diagram of Block at Impending Motion (Minimum Spring Tension)



$F_m = \mu_s N$

$$\xrightarrow{+} \sum F_x = 0$$

$$\mu_s N - (100 \text{ lb}) \sin 30° + T_{min} = 0$$

$$T_{min} = -\mu_s N + (100 \text{ lb}) \sin 30°$$

Solve Equations of Equilibrium

1. $$N = (100\ \text{lb})\cos 30°$$

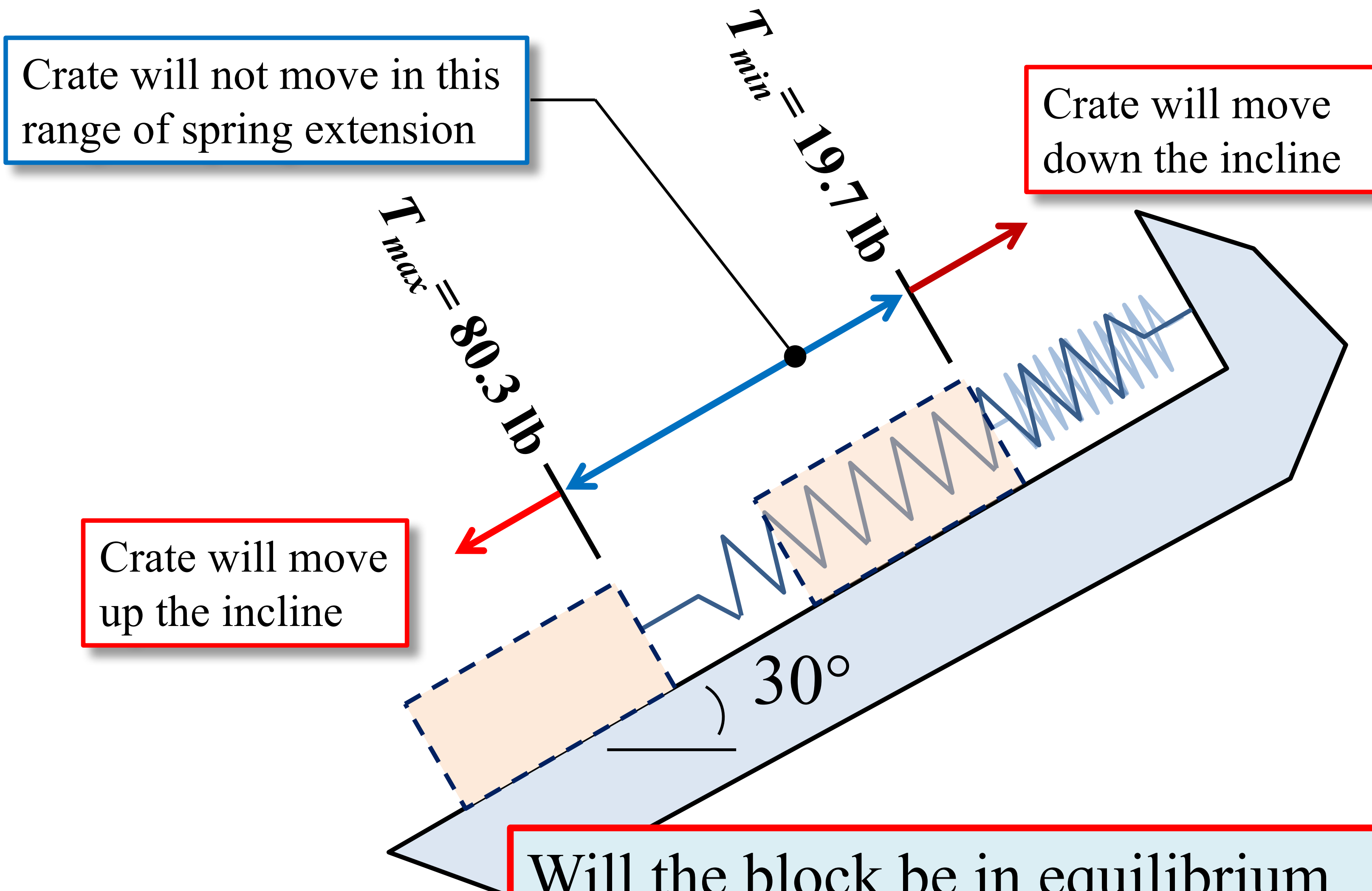
2. $$T_{min} = -\mu_s N + (100\ \text{lb})\sin 30°$$

$$T_{min} = -\mu_s(100\ \text{lb})\cos 30° + (100\ \text{lb})\sin 30°$$

$\mu_s = 0.35$

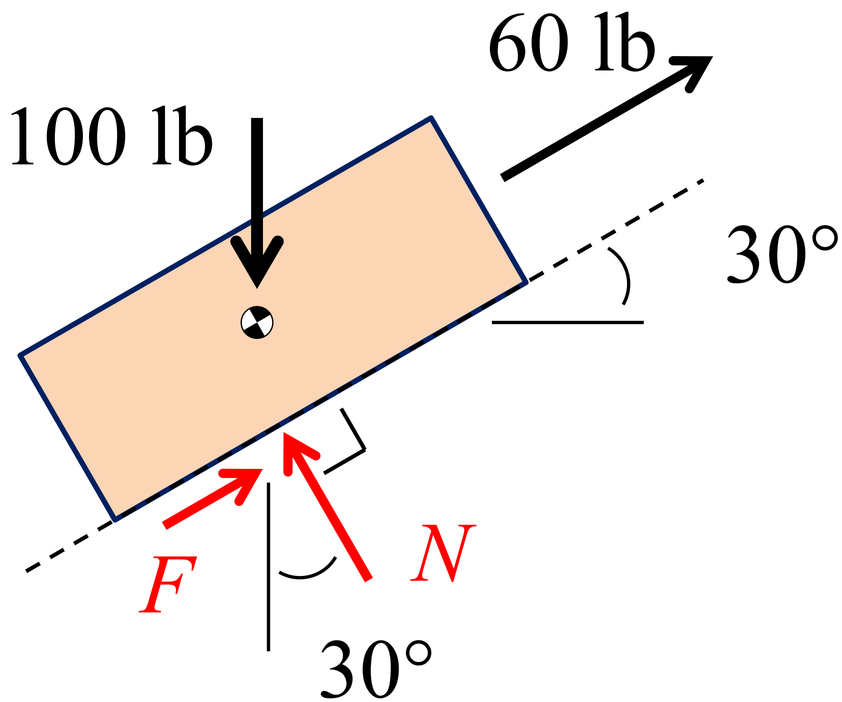
$$T_{min} = 19.7\ \text{lb}$$

Summary of Previous Results



Will the block be in equilibrium when the spring tension is 60 lb?

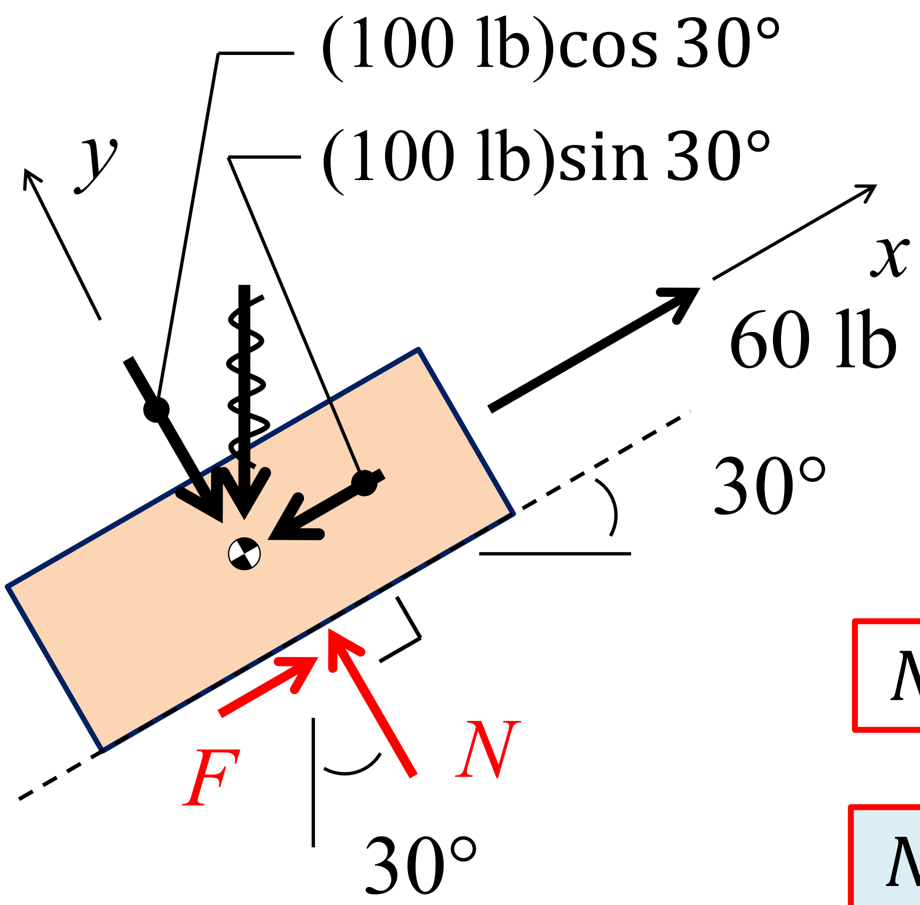
Free-Body Diagram of Block When Spring Tension is 60 lb



Notes

- Sense of friction force is assumed;
- Three forces on block – must be concurrent (i.e. two equations of equilibrium available to solve for unknown forces;
- Convenient to orient reference coordinate system inclined with the surface;
- Two unknowns (*N*, *F*).

Free-Body Diagram of Block When Spring Tension is 60 lb



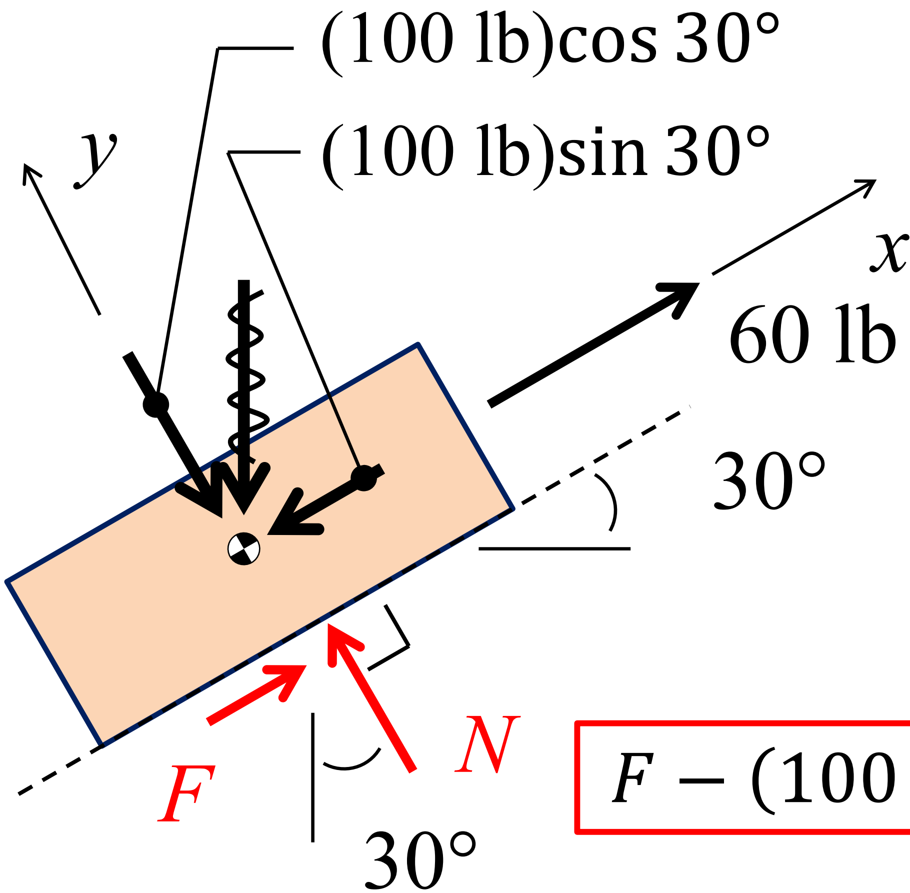
10 lb

$$+\uparrow \sum F_y = 0$$

$$N - (100\text{ lb})\cos 30° = 0$$

$$N = (100\text{ lb})\cos 30°$$

Free-Body Diagram of Block When Spring Tension is 60 lb



$$\xrightarrow{+} \sum F_x = 0$$

$$F - (100\text{ lb})\sin 30° + 60\text{ lb} = 0$$

$$F = -10\text{ lb}$$

Sense of friction force is opposite assumed

Free-Body Diagram of Block When Spring Tension is 60 lb Showing Results