Distributed Loads on Beams

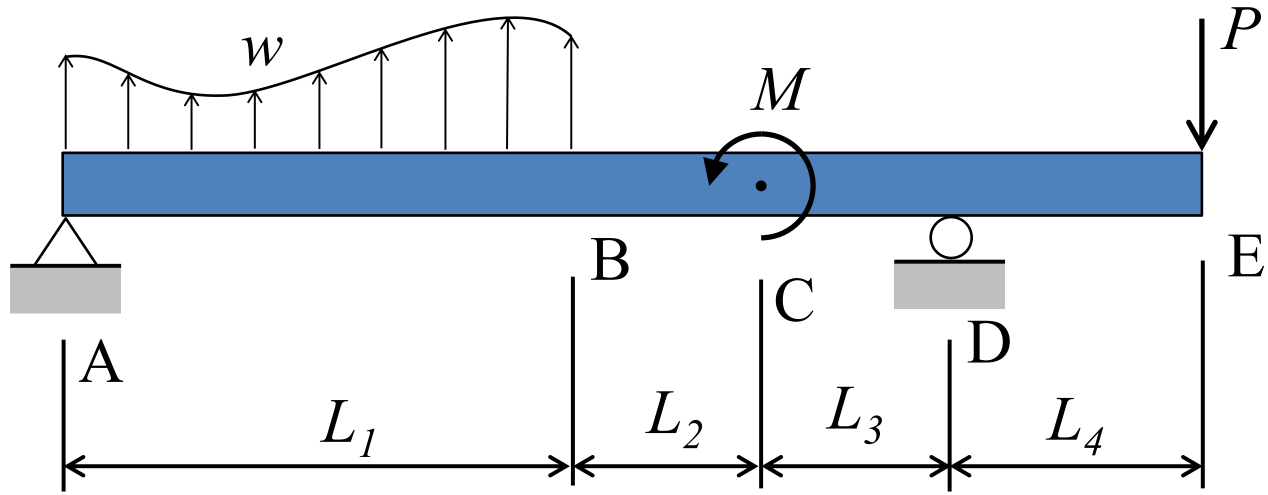
Steven Vukazich
San Jose State University

All Loads are Distributed Over a Finite Contact Length or Area

The actual contact pressure between the road and the tire is distributed over the contact length, L_C.

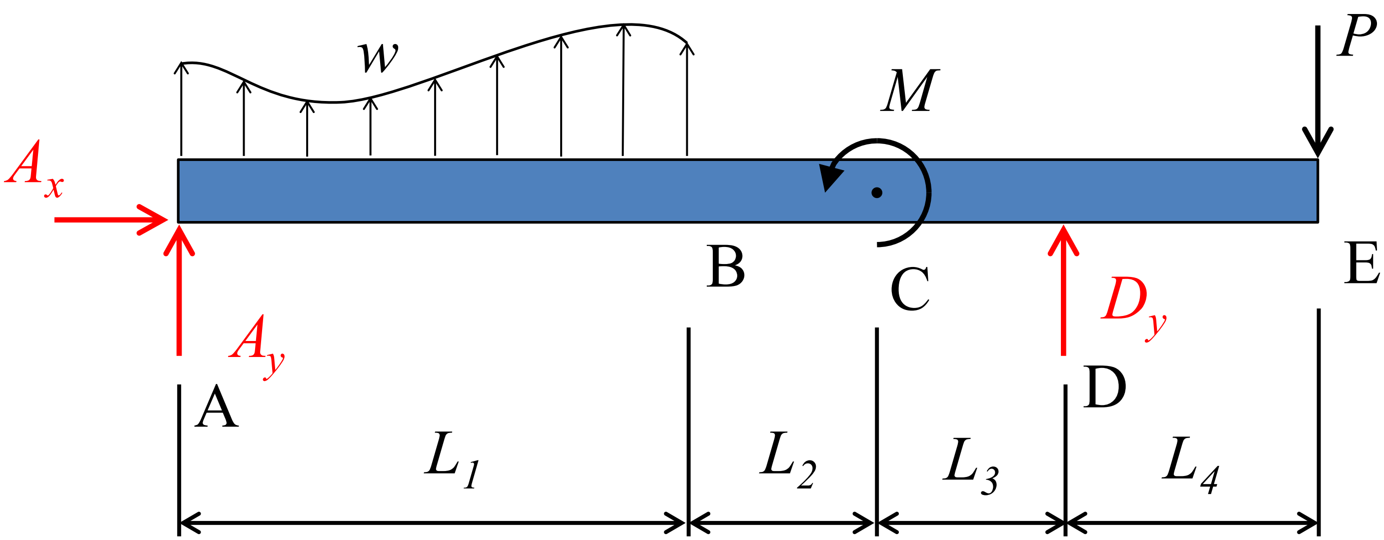
For overall equilibrium calculations, we can replace the distributed load with an equivalent point load, R

Consider the Beam with the Supports and Loading



We would like to find the support reactions at the pin at A and roller at D

Free Body Diagram of the Beam



$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0$

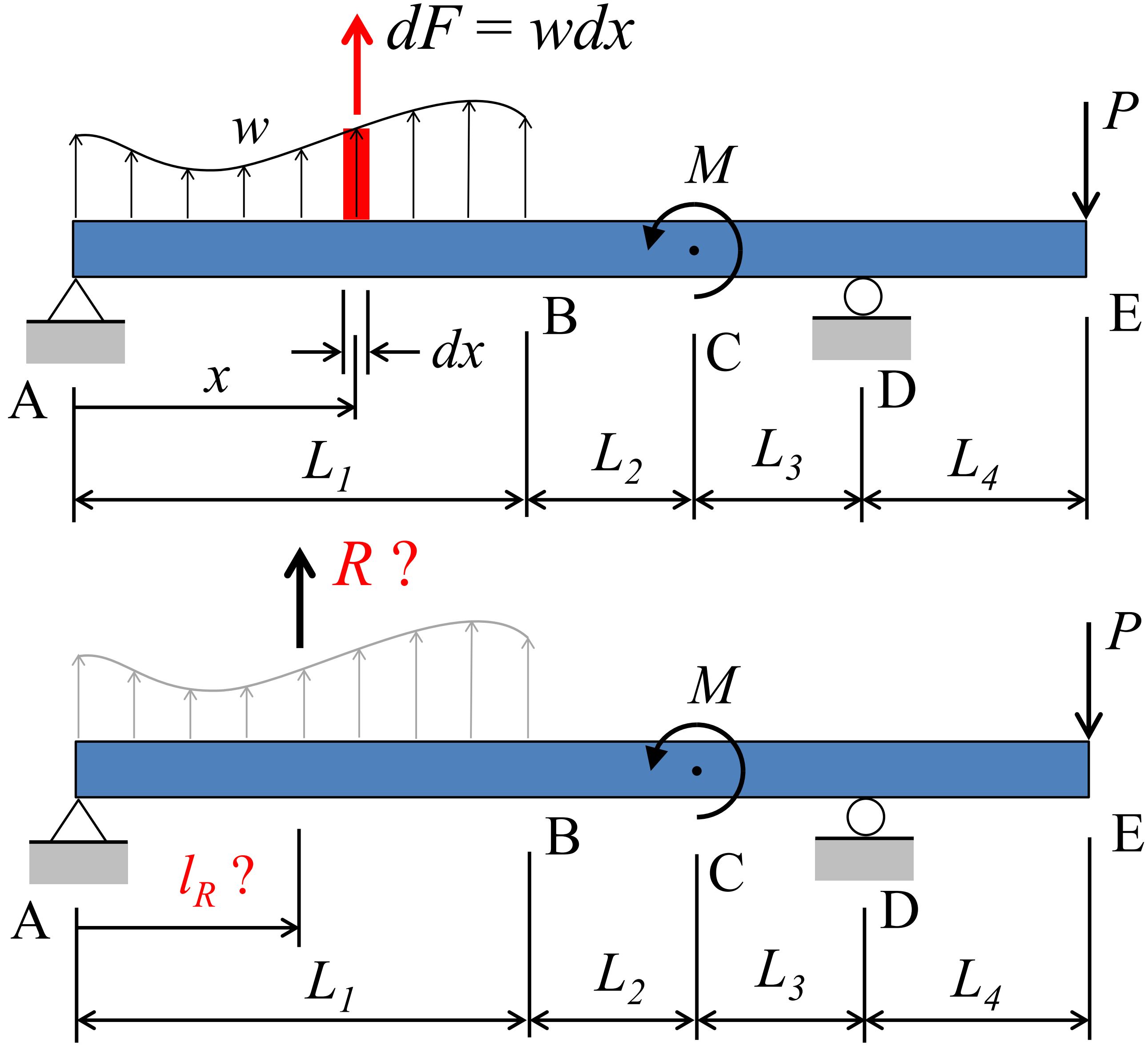
$\circlearrowleft + \sum M_A = 0 \rightarrow ?$

$+\uparrow \sum F_y = 0 \rightarrow ?$

How can we account for the effect of the distributed load?

We need to find a point load that is equivalent to the distributed load

Find an Equivalent Point Load, R



Find the Equivalent Point Load Force

For the load to be equivalent, it is required to:

1. Produce the same force as the distributed load;
2. Produce the same moment as the distributed load.

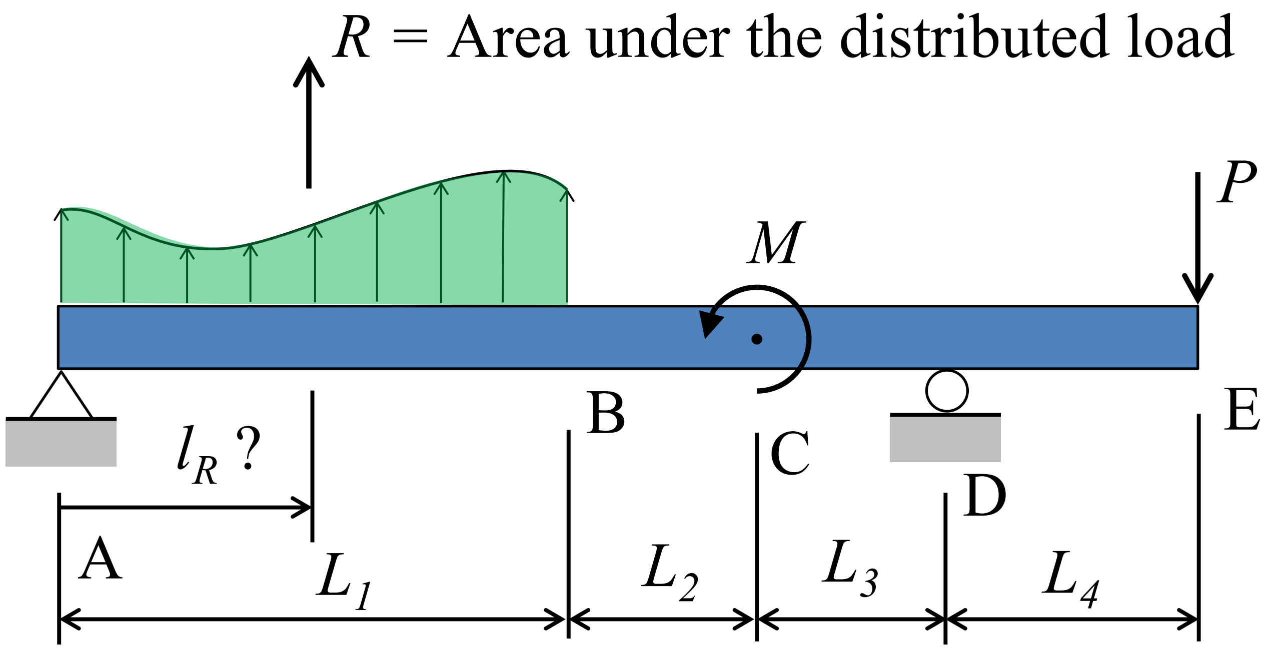
Equivalent Force

$$R = \int_A^B dF$$

$$R = \int_A^B w\,dx$$

Resultant force is equal to the area under the distributed load

Find the Location of the Equivalent Point Load



Where would we place the Equivalent point load to produce the same moment as the distributed load?

Find the Location of the Equivalent Point Load

For the load to be equivalent, it would have to produce the same moment as the distributed load

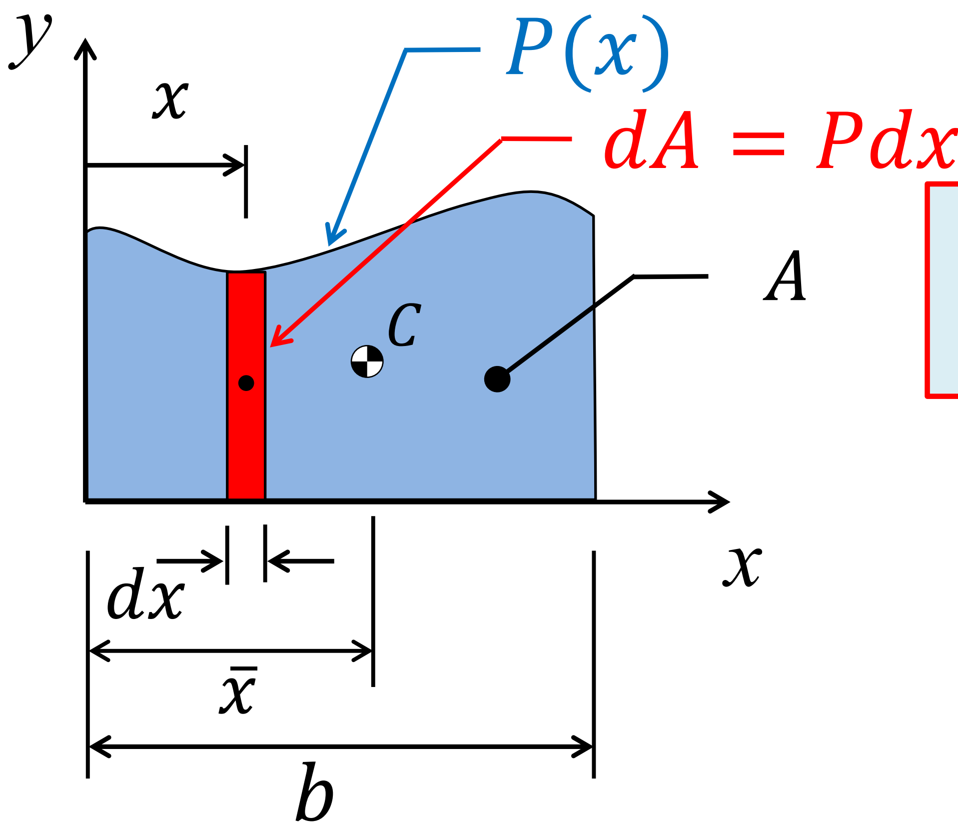
Equivalent moment about point A

$$Rl_R = \int_A^B x\,dF = \int_A^B xw\,dx$$

$$Rl_R = \int_A^B xw\,dx$$

Moment of the resultant force is analogous to the definition of the first moment of an area

Recall the First Moment of an Area Cut into Vertical Strips



$$A\bar{x} = \int_0^b x\, dA = \int_0^b x\, P dx$$

$$R l_R = \int_A^B x w dx$$

$A \rightarrow R$

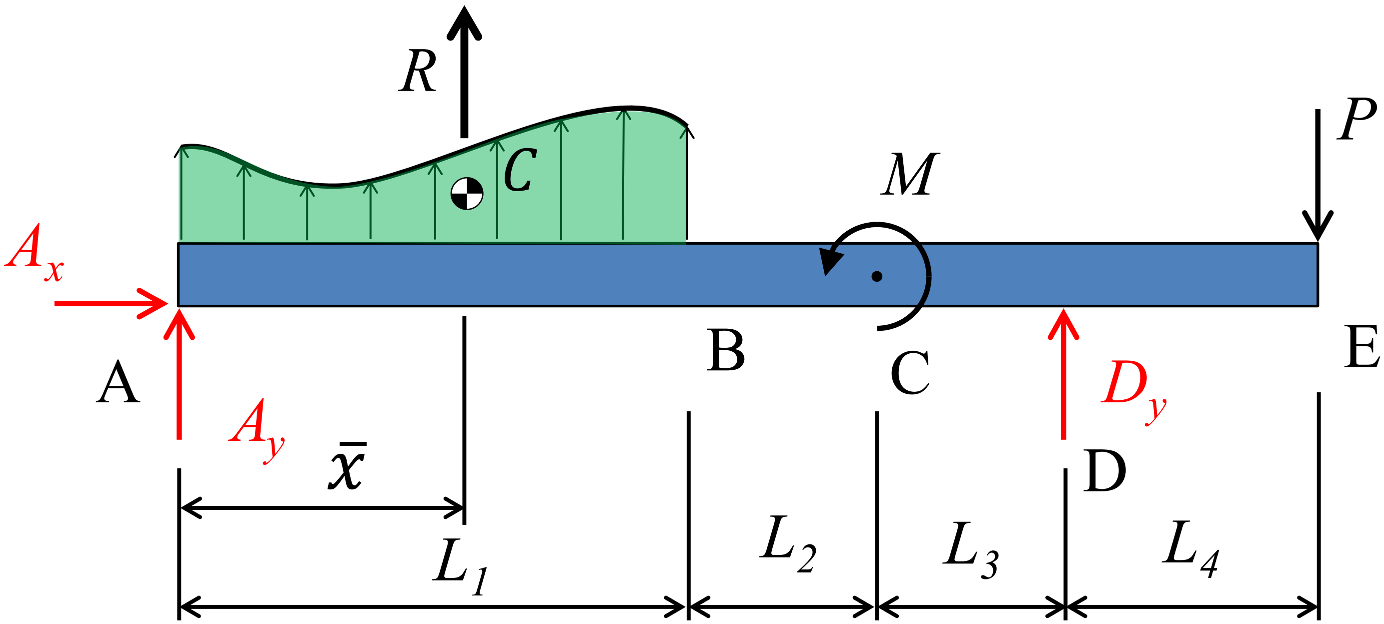
$P(x) \rightarrow w(x)$

Resultant force acts at the centroid of the area of the distributed load

Therefore,

$\bar{x} \rightarrow l_R$

Summary



$$\overset{+}{\rightarrow} \sum F_x = 0 \rightarrow A_x = 0$$

R = Area under the distributed load
$\bar{x}$ = Distance to the centroid of the area defined by the distributed load

$$\circlearrowleft + \sum M_A = 0 \searrow$$

$$R\bar{x} + M + D_y(L_1 + L_2 + L_3) - P(L_1 + L_2 + L_3 + L_4) = 0$$

$$+\uparrow \sum F_y = 0 \rightarrow A_y - R + D_y - P = 0$$