# Analysis of Statically Indeterminate Structures Using the Force Method

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# Statically Indeterminate Structures

At the beginning of the course, we learned that a **stable structure** that contains **more unknowns than independent equations of equilibrium** is **Statically Indeterminate.**

## Advantages

- Redundancy (several members must fail for the structure to become unstable);
- Often maximum stresses is certain members are reduced;
- Usually deflections are reduced.

## Disadvantages

- Connections are often more expensive;
- Finding forces and deflections using hand analysis is much more complicated.

# Steps in Solving an Indeterminate Structure using the Force Method

Determine degree of Indeterminacy
Let *n* =degree of indeterminacy
(i.e. the structure is indeterminate to the nth degree)

Chapter 3

Define Primary Structure and the *n* Redundants

Define the Primary Problem

Solve for the *n* Relevant Deflections in Primary Problem

Chapters 3,4,5 then 7 or 8

Define the *n* Redundant Problems

Solve for the *n* Relevant Deflections in each Redundant Problem

Chapters 3,4,5 then 7 or 8

Write the *n* Compatibility Equations at Relevant Points

Solve the *n* Compatibility Equations to find the *n* Redundants

Use the Equations of Equilibrium to solve for the remaining unknowns

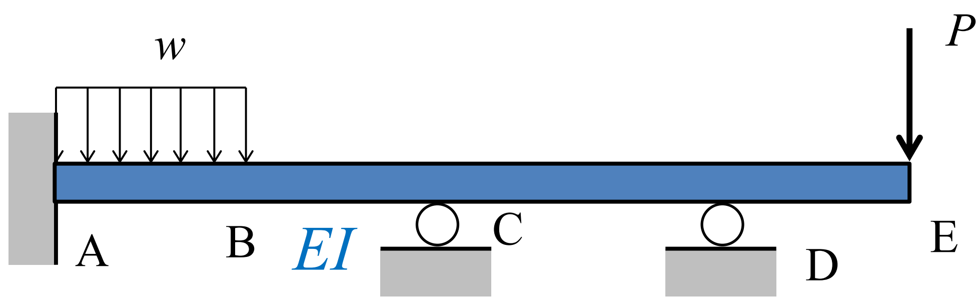
Chapter 3

Construct Internal Force Diagrams (if necessary)

Chapters 3,4,5

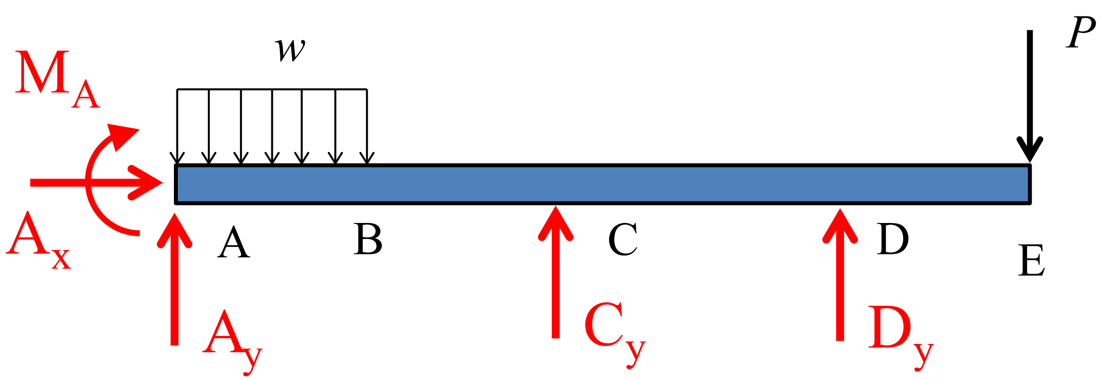
# Force Method of Analysis

Consider the beam




Beam is stable

FBD




$X = 5$

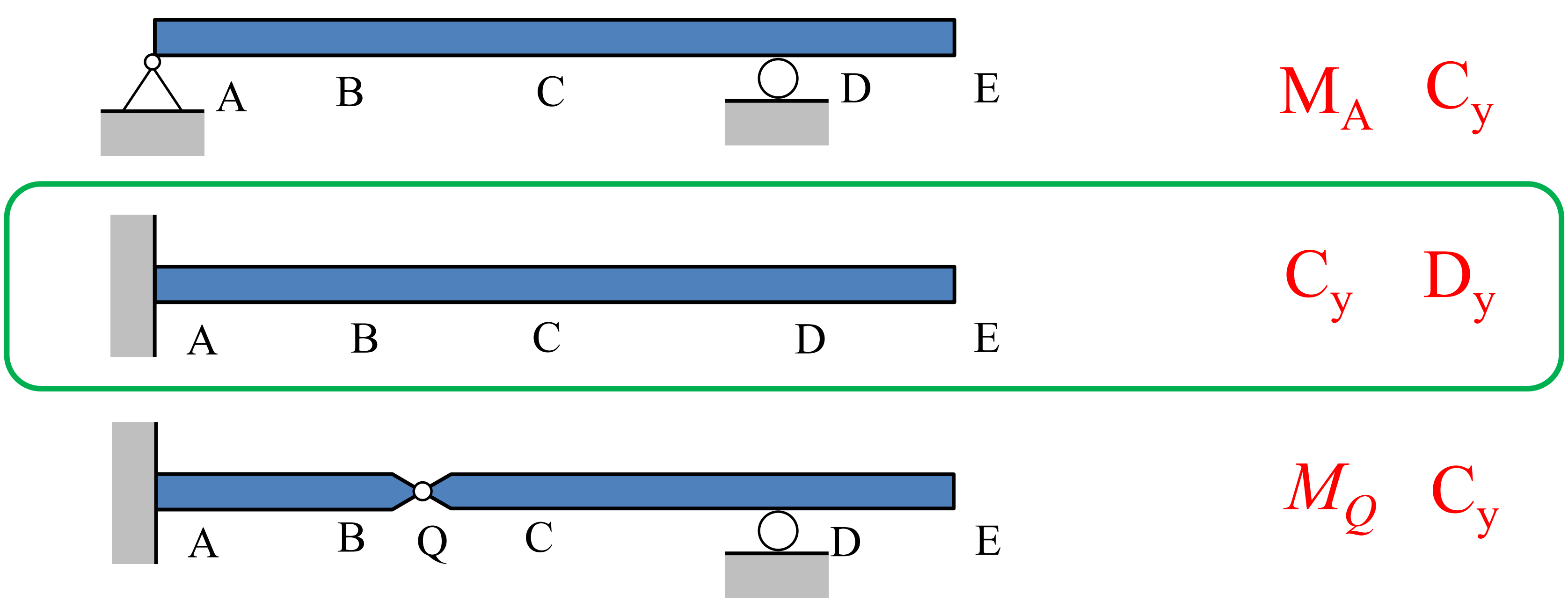
$3n = 3(1) = 3$

Statically Indeterminate to the 2nd degree

# Define Primary Structure and Redundants

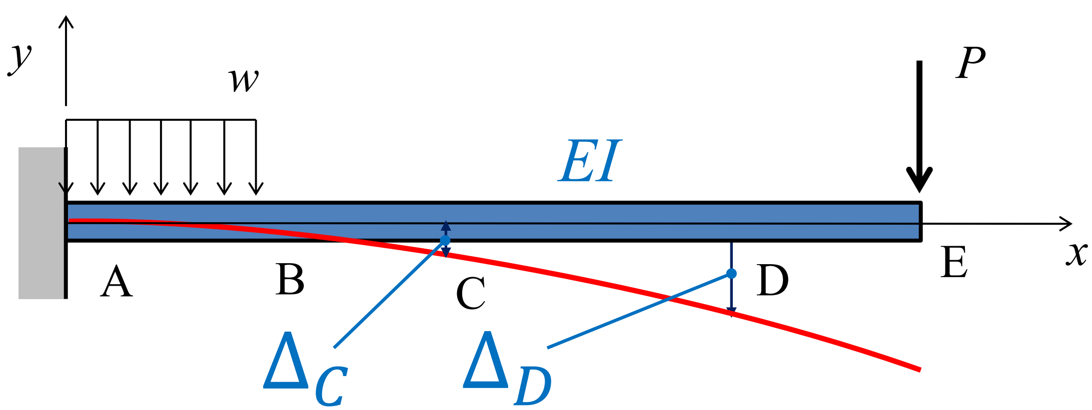
- Remove all applied loads from the actual structure;
- Remove support reactions or internal forces to define a primary structure;
- Removed reactions or internal forces are called redundants;
- Same number of redundants as degree of indeterminacy
- Primary structure must be stable and statically determinate;
- Primary structure is not unique – there are several choices.

**Primary Structure** | **Redundants**



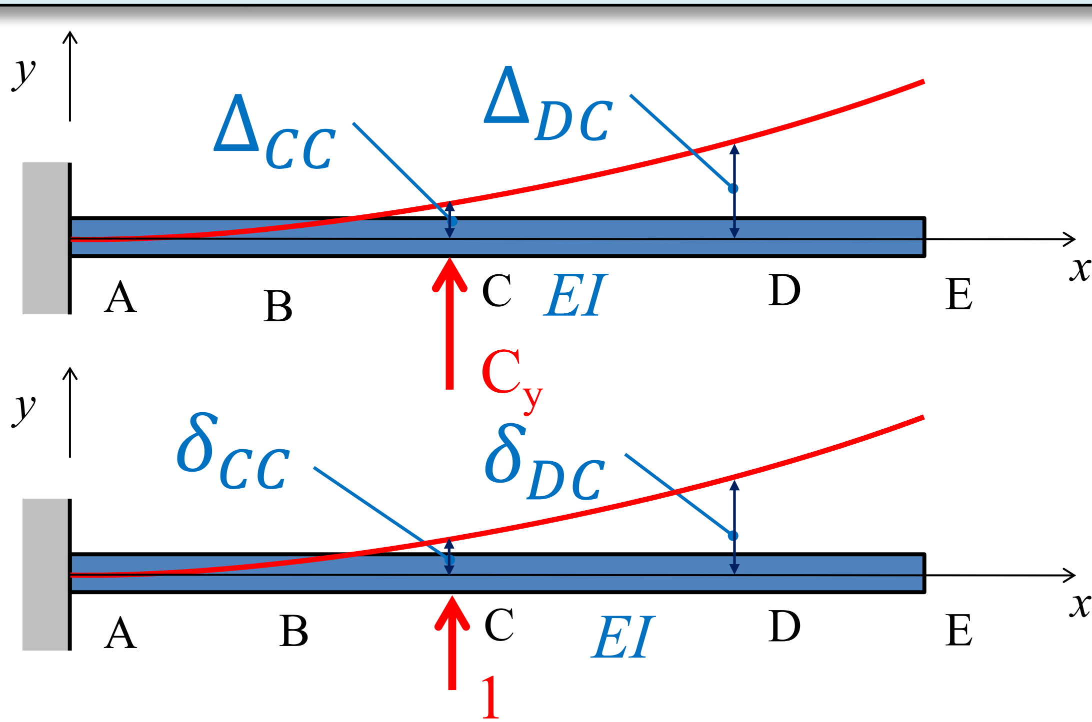
# Define and Solve the Primary Problem

- Apply all loads on actual structure to the primary structure;
- Define a reference coordinate system;
- Calculate relevant deflections at points where redundants were removed.



# Define and Solve the Redundant Problems

- There are the same number of redundant problems as degrees of indeterminacy;
- Define a reference coordinate system;
- Apply only one redundant to the primary structure;
- Write the redundant deflection in terms of the flexibility coefficient and the redundant for each redundant problem.
- Calculate the flexibility coefficient associated with the relevant deflections for each redundant problem;

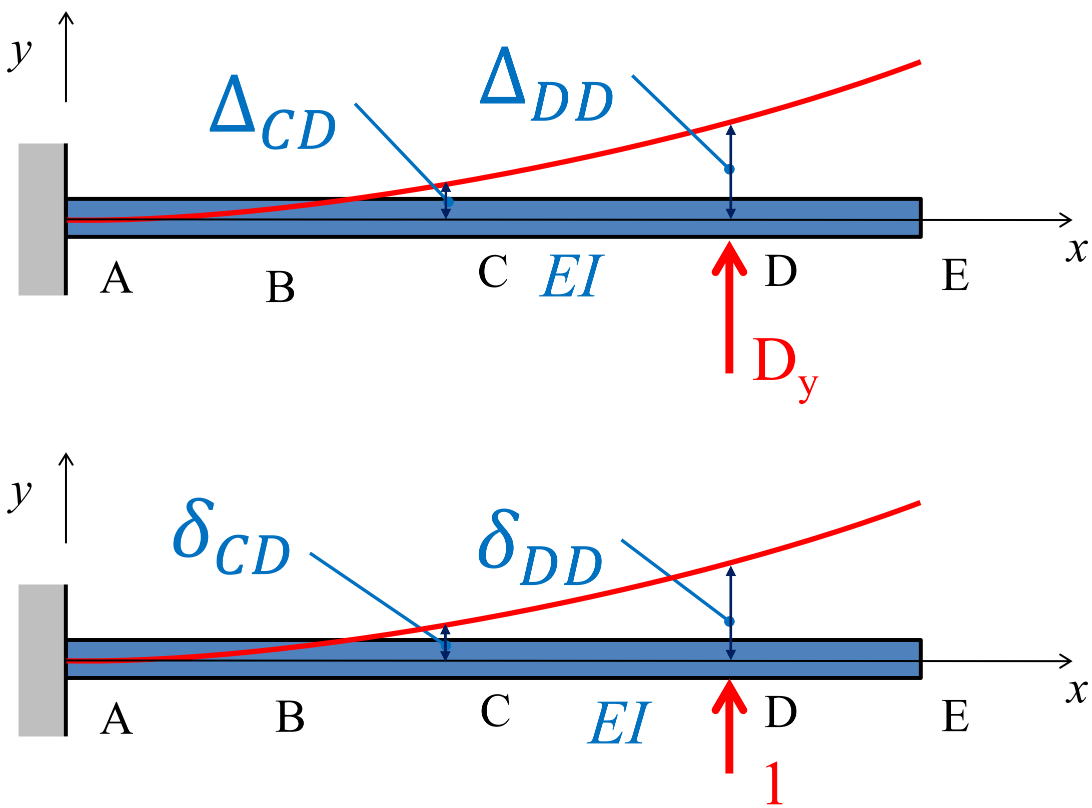



**Redundant Problem 1**

$$\Delta_{CC} = C_y \delta_{CC}$$

$$\Delta_{DC} = C_y \delta_{DC}$$

# Define and Solve the Redundant Problems




**Redundant Problem 2**

$$\Delta_{CD} = D_y \delta_{CD}$$

$$\Delta_{DD} = D_y \delta_{DD}$$

# Compatibility Equations

**Compatibility at Point C**

$$\Delta_C + \Delta_{CC} + \Delta_{CD} = 0$$

**Compatibility at Point D**

$$\Delta_D + \Delta_{DC} + \Delta_{DD} = 0$$

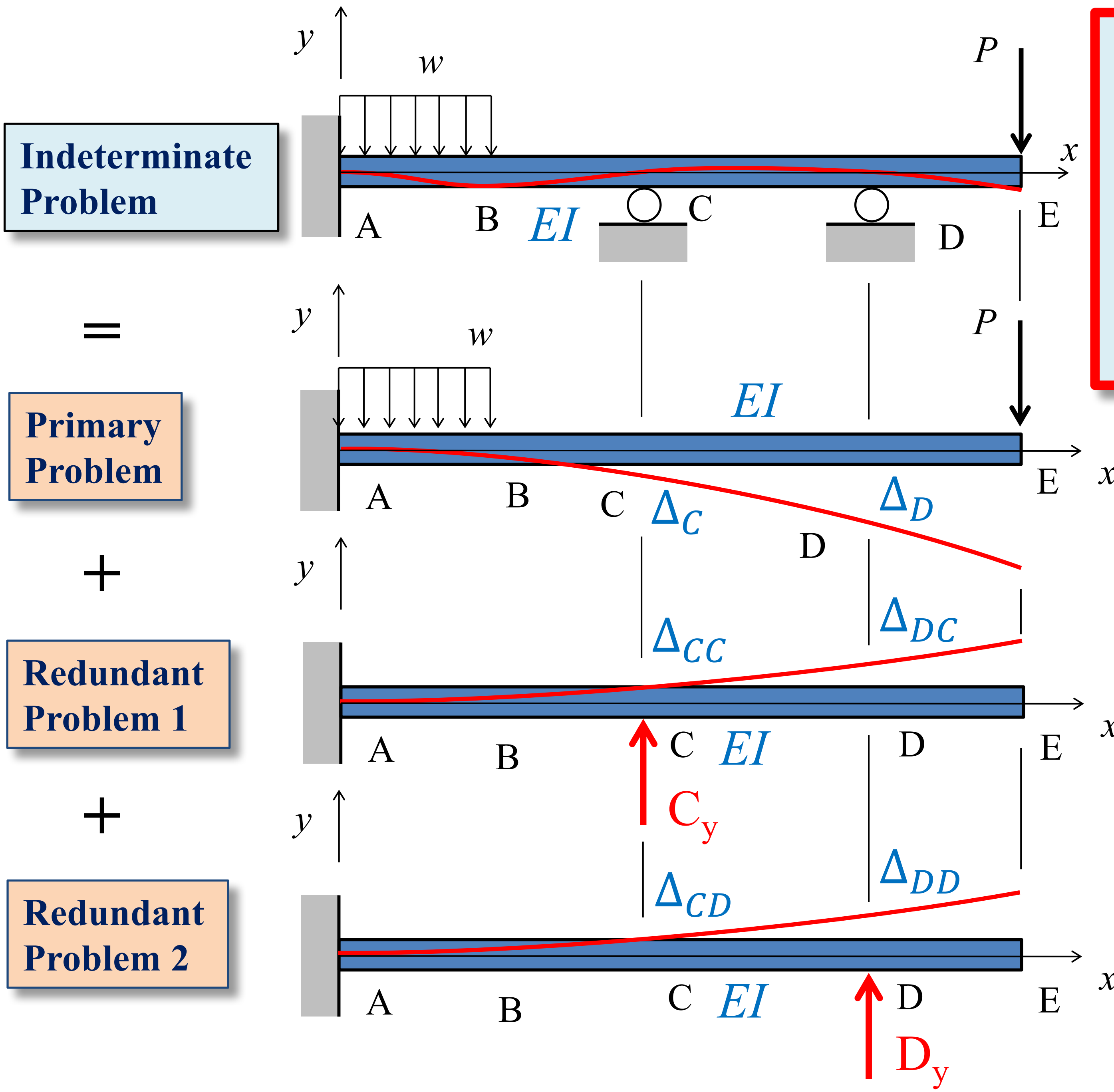
**Compatibility Equations in terms of Redundants and Flexibility Coefficients**

$$\Delta_C + C_y\delta_{CC} + D_y\delta_{CD} = 0$$

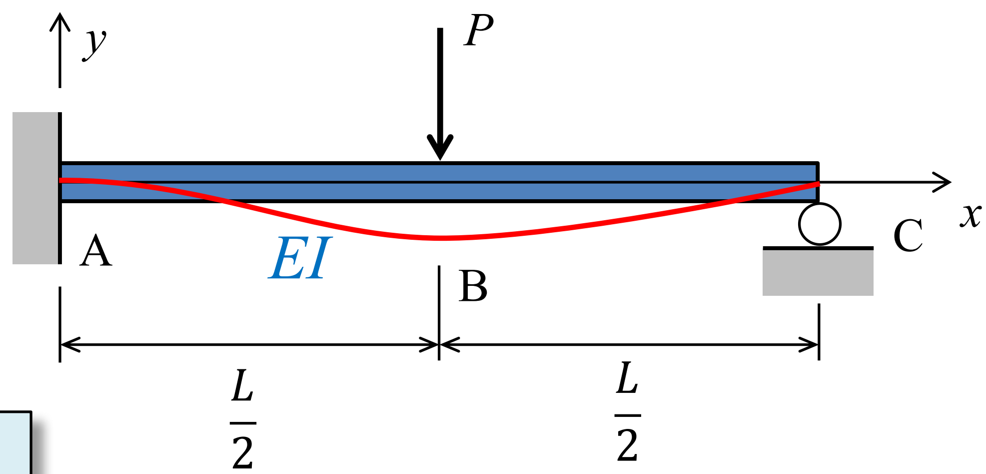
$$\Delta_D + C_y\delta_{DC} + D_y\delta_{DD} = 0$$

**Solve for $C_y$ and $D_y$**

# The Force Method is Based on the Principle of Superposition

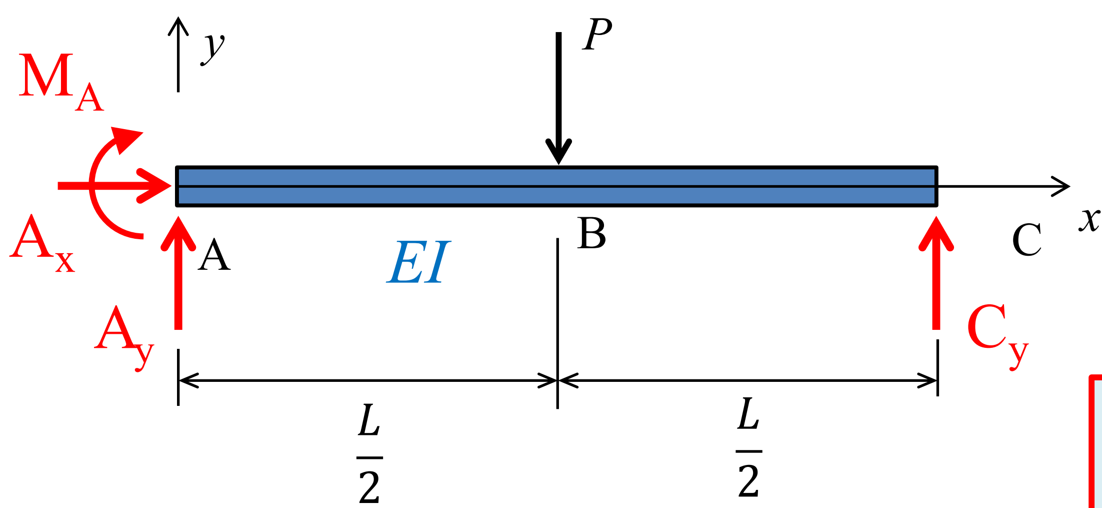


# Example Problem




For the indeterminate beam subject to the point load, $P$, find the support reactions at A and C. $EI$ is constant.

FBD




Beam is stable

$X = 4$

$3n = 3(1) = 3$

Statically Indeterminate to the 1st degree

# Define Primary Structure and Redundant

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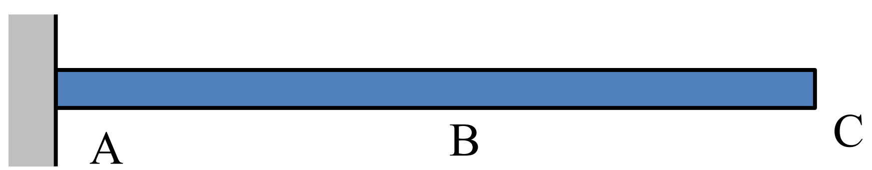
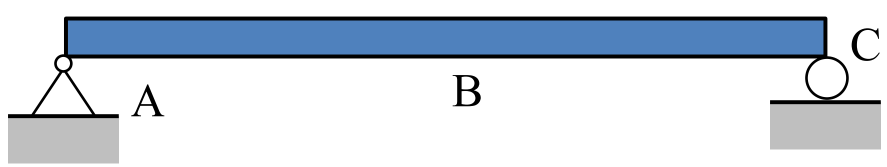
**Primary Structure**

**Redundant**




$C_y$




$M_A$

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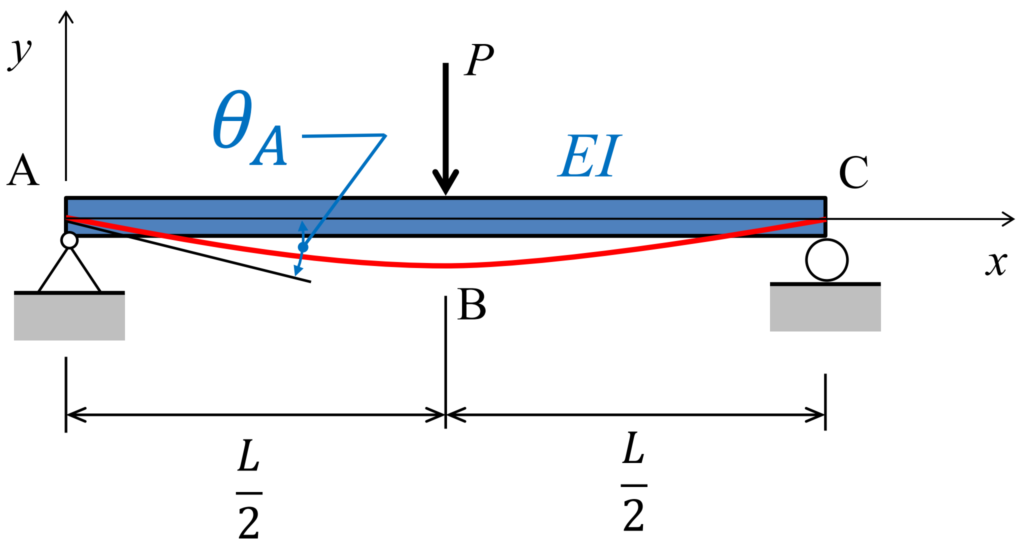
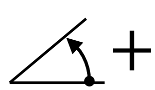



**From Tabulated Solutions**




Counter-clockwise rotations positive

$$\theta_A = -\frac{PL^2}{16EI}$$

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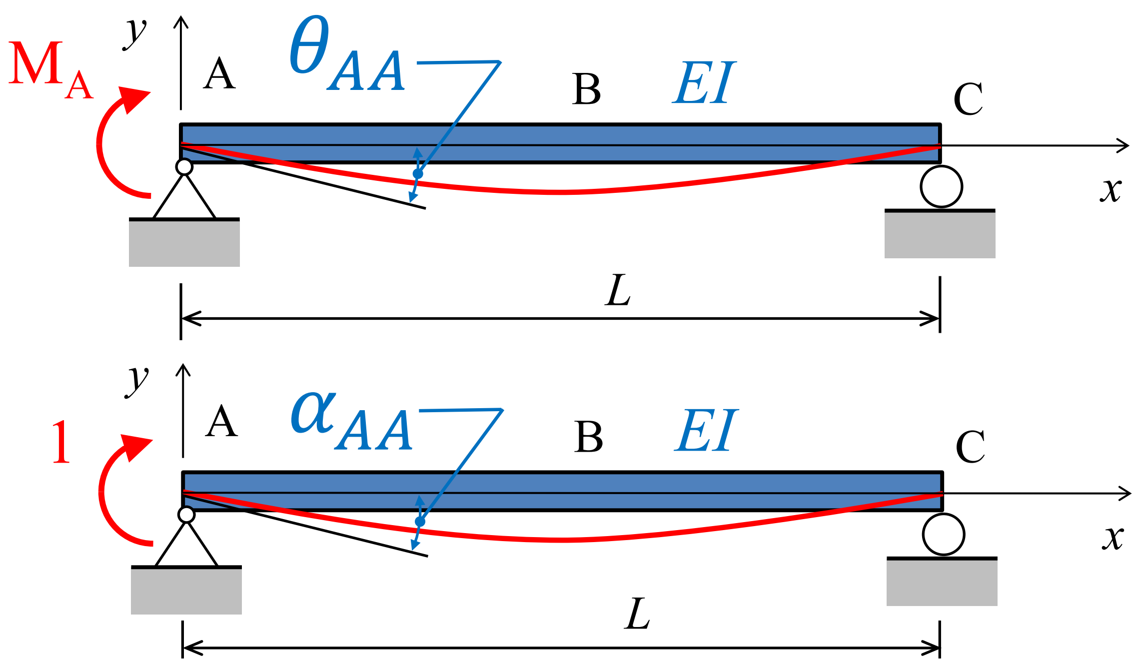



**Redundant Problem**

$$\theta_{AA} = M_A \alpha_{AA}$$

**From Tabulated Solutions**

$$\alpha_{AA} = -\frac{L}{3EI}$$

# Compatibility Equation at Point A

**Compatibility at Point A**

$$\theta_A + \theta_{AA} = 0$$

**Compatibility Equation in terms of Redundant and Flexibility Coefficient**

$$\theta_A + M_A \alpha_{AA} = 0$$

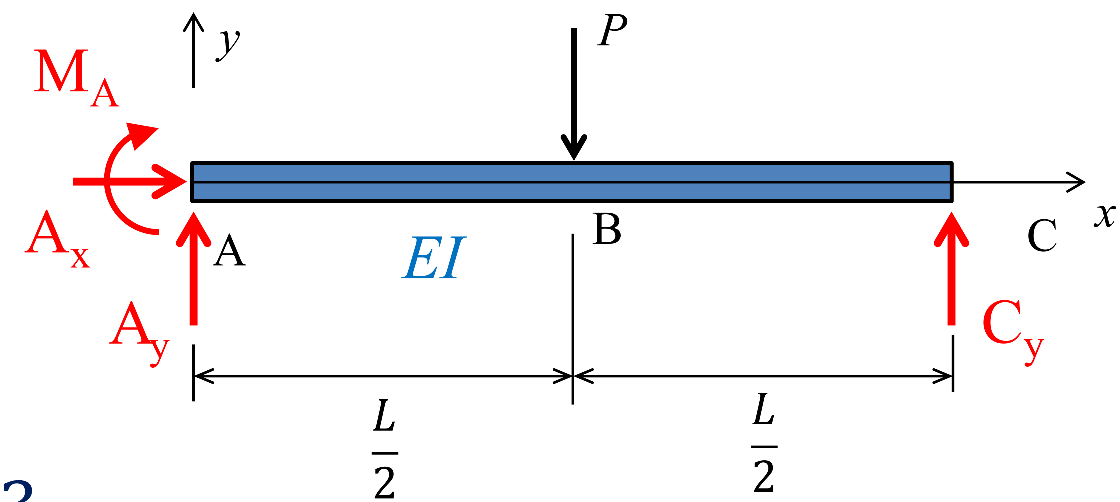
$$-\frac{PL^2}{16EI} + M_A\left(-\frac{L}{3EI}\right) = 0$$

**Solve for $M_A$**

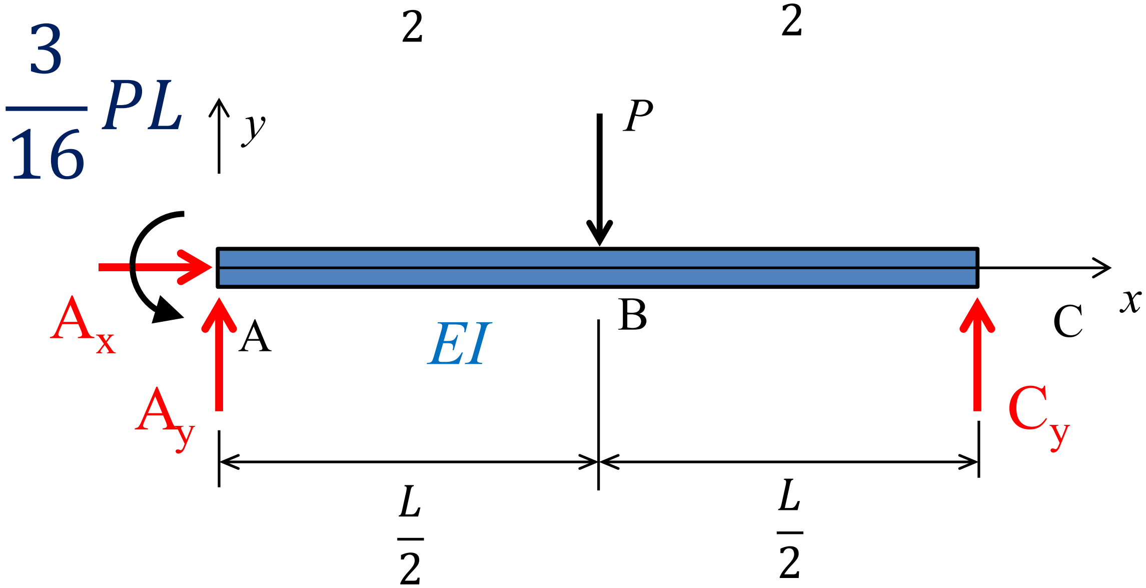
$$M_A = \frac{PL^2}{16EI}\left(-\frac{3EI}{L}\right)$$

$$M_A = -\frac{3}{16}PL$$

# Free Body Diagram

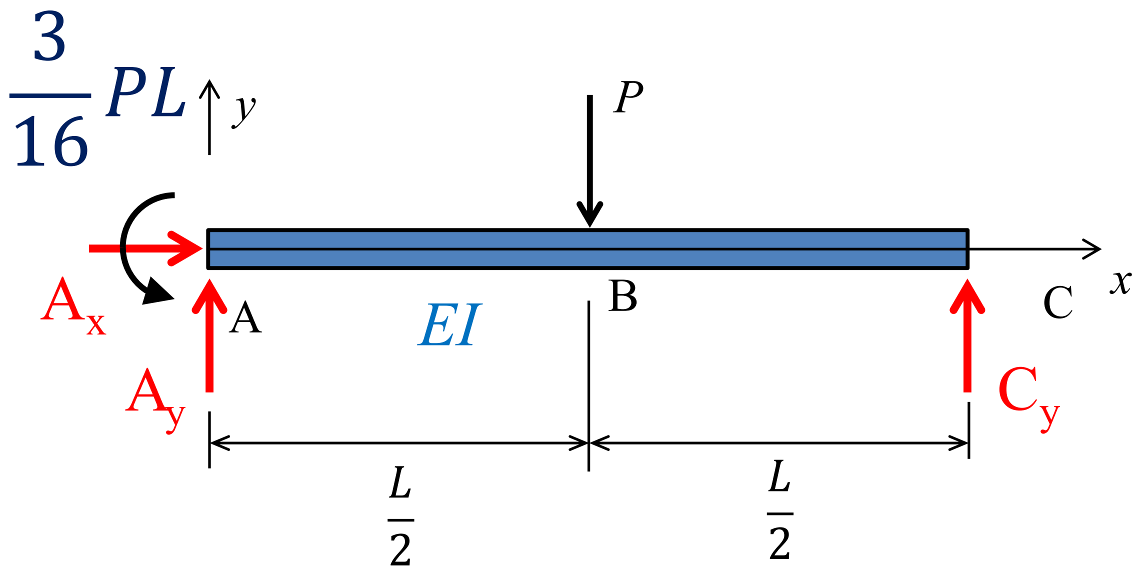



$$M_A = -\frac{3}{16}PL$$




Can now use equilibrium equations to find the remaining three unknowns

# Find Remaining Unknowns



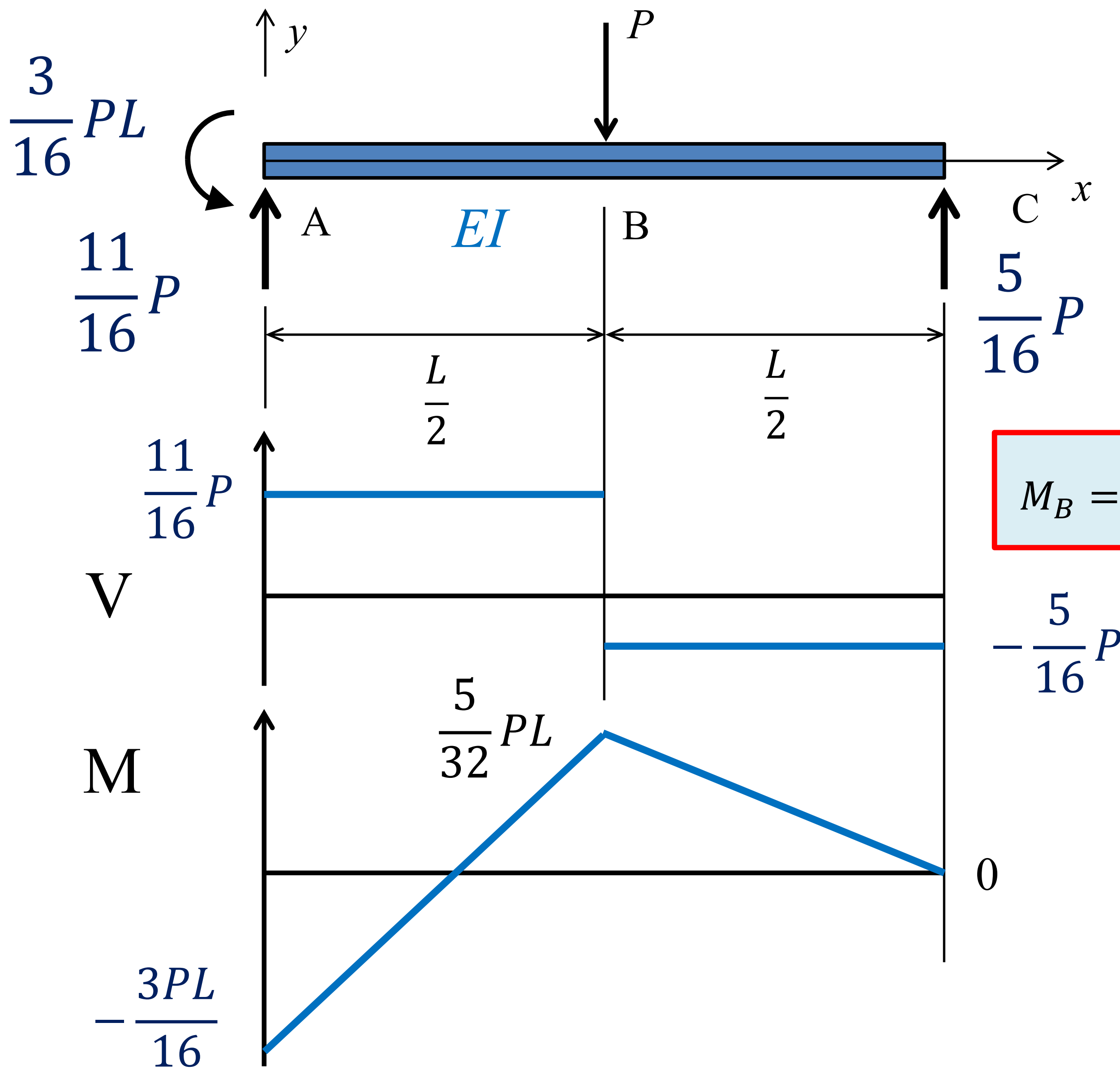

Can now use equilibrium equations to find the remaining three unknowns

$$\xrightarrow{+} \sum F_x = 0 \rightarrow A_x = 0$$

$$\circlearrowleft + \sum M_A = 0 \rightarrow C_y = \frac{5}{16}P$$

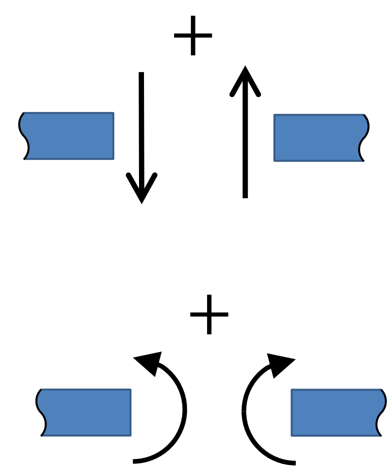
$$+\uparrow \sum F_y = 0 \rightarrow A_y = \frac{11}{16}P$$

# Draw V and M Diagrams of the Beam




$$M_B - M_A = \left(\frac{11}{16}P\right)\left(\frac{L}{2}\right)$$

$$M_B = -\frac{3}{16}PL + \frac{11}{32}PL = \frac{5}{32}PL$$



# Superposition of Primary and Redundant Problems