Method of Virtual Work for Trusses

Temperature Change and Fabrication Errors

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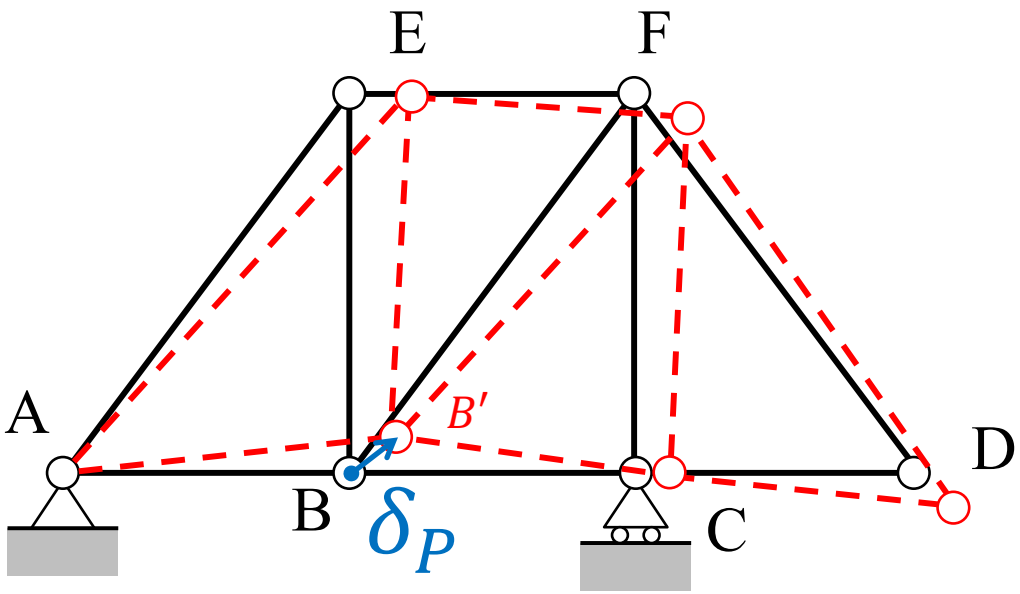
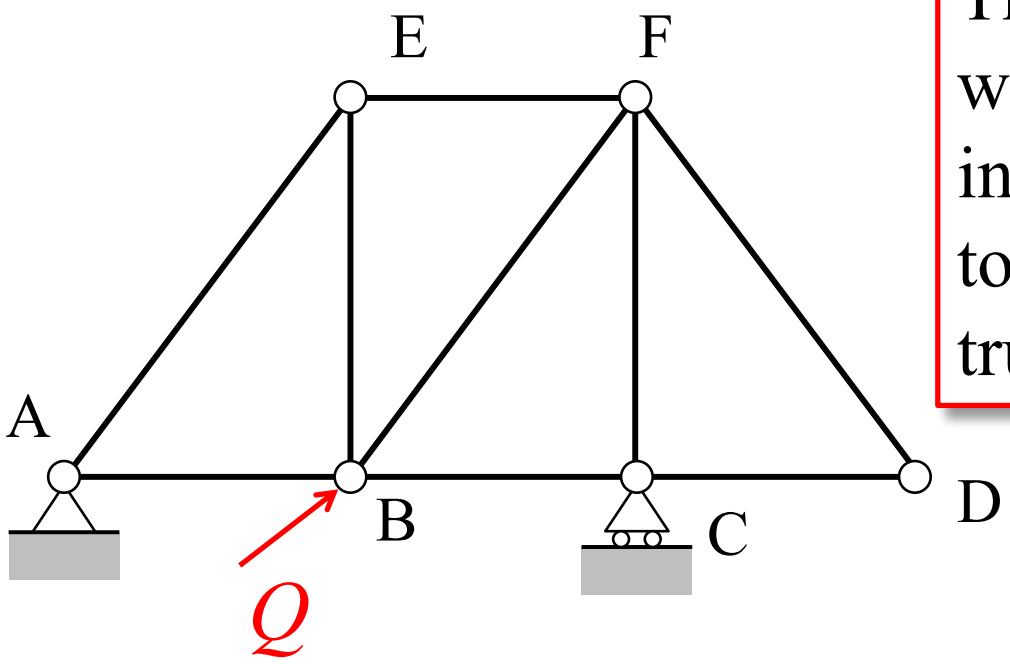
Consider a Truss Structure Subjected To Temperature Change of Some or All Members



A_i = Cross sectional area
E_i = Modulus of Elasticity
L_i = Length of truss member
α_i = Coefficient of thermal expansion of truss member
n_T = Total number of truss members subjected to temperature change

We want to find the deflection of joint B due to temperature changes in n_T truss members

Apply Virtual Force

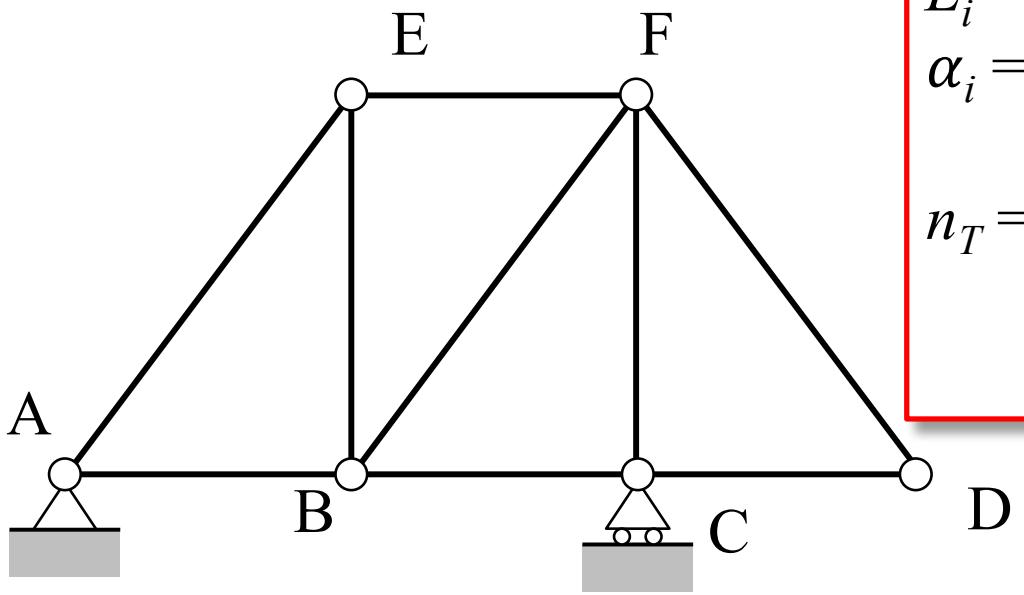


The virtual force will cause an internal axial force to develop in each truss member, F_{Qi}

Apply a virtual force **in-line** with the real displacement δ_P

$$W_Q = Q\delta_P$$

Real Deformation due to Temperature Changes in Truss Members



L_i = Length of truss member
α_i = Coefficient of thermal expansion of truss member
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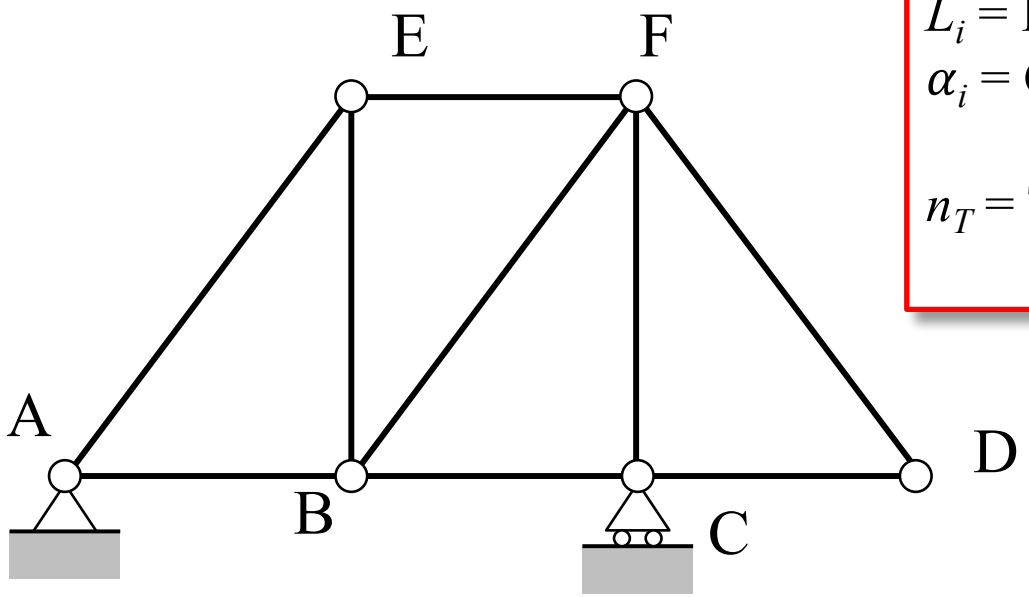
The real temperature change causes an axial deformation of each truss member, $\Delta L_{Pi} = \alpha_i \Delta T_i L_i$

Virtual Strain Energy



L_i = Length of truss member
α_i = Coefficient of thermal expansion of truss member
n_T = Total number of truss members subjected to temperature change

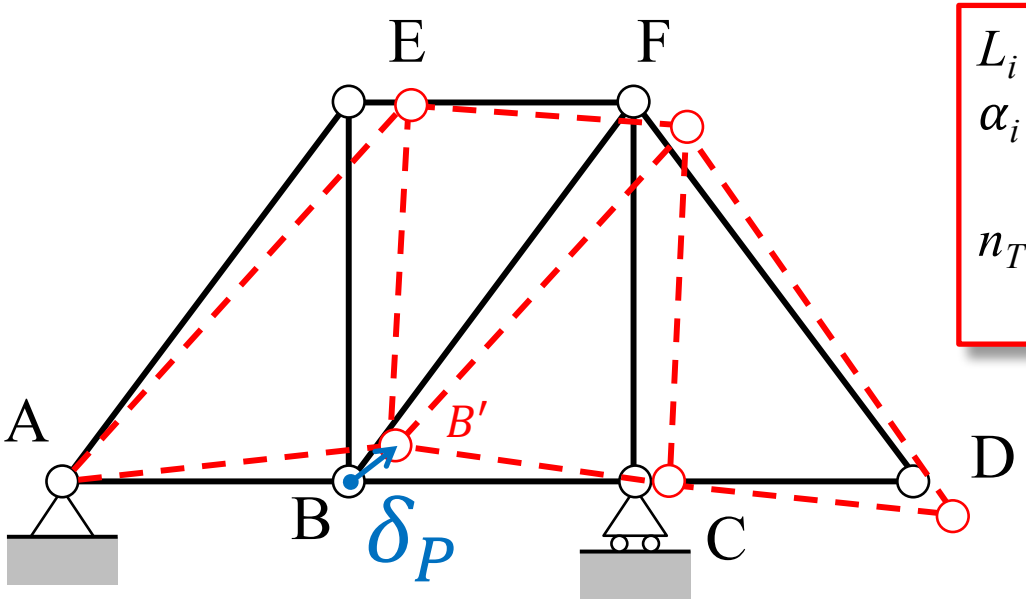
Virtual strain energy developed in an individual truss member

$$U_{Qi} = F_{Qi} \cdot \Delta L_{Pi} = F_{Qi}\alpha_i \Delta T_i L_i$$

Virtual strain energy for the entire truss

$$U_Q = \sum_{i=1}^{n_T} F_{Qi}\alpha_i \Delta T_i L_i$$

Principle of Virtual Work for Truss Deflections Due to Temperature Changes



L_i = Length of truss member
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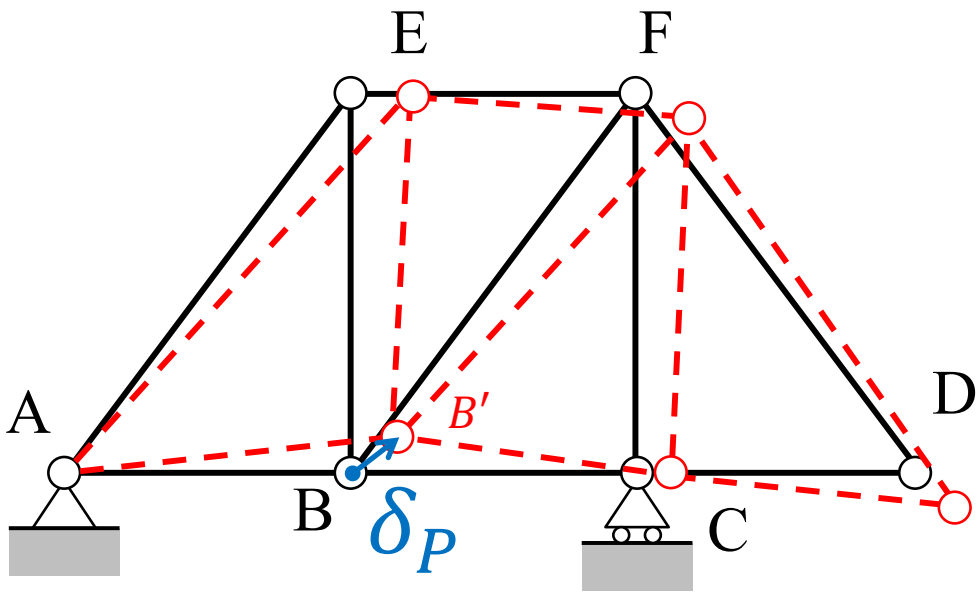
$$W_Q = U_Q$$

$$Q\delta_P = \sum_{i=1}^{n_T} F_{Qi}\alpha_i \Delta T_i L_i$$

Real Deformation

Virtual Loads

Procedure For Virtual Work Deflection Analysis



A_i = Cross sectional area
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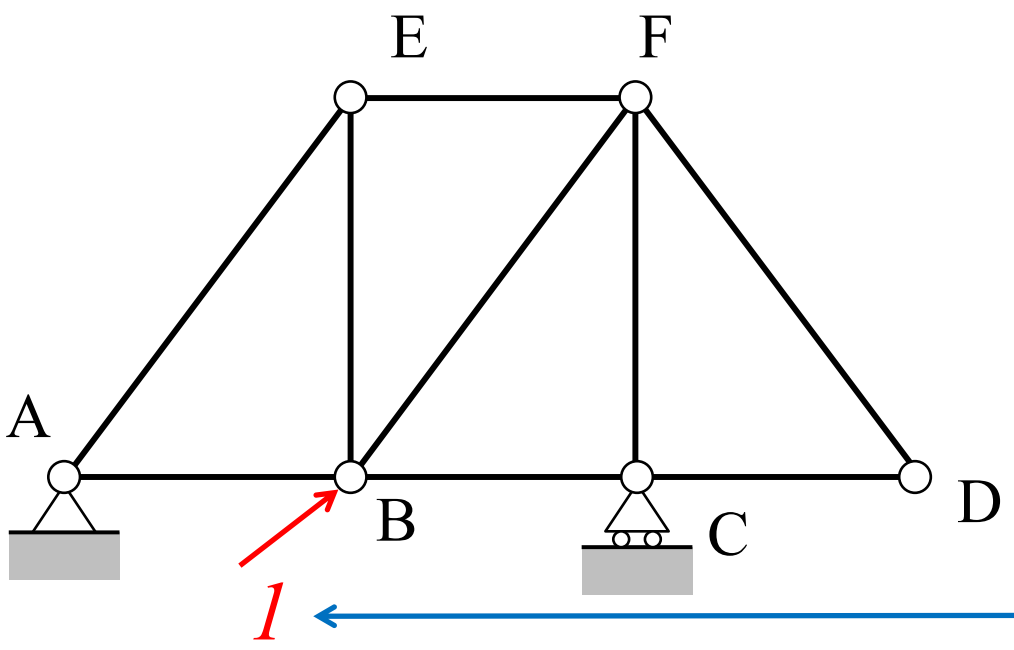
We want to find the real deflection of joint B due to the temperature change of n_T members, δ_P

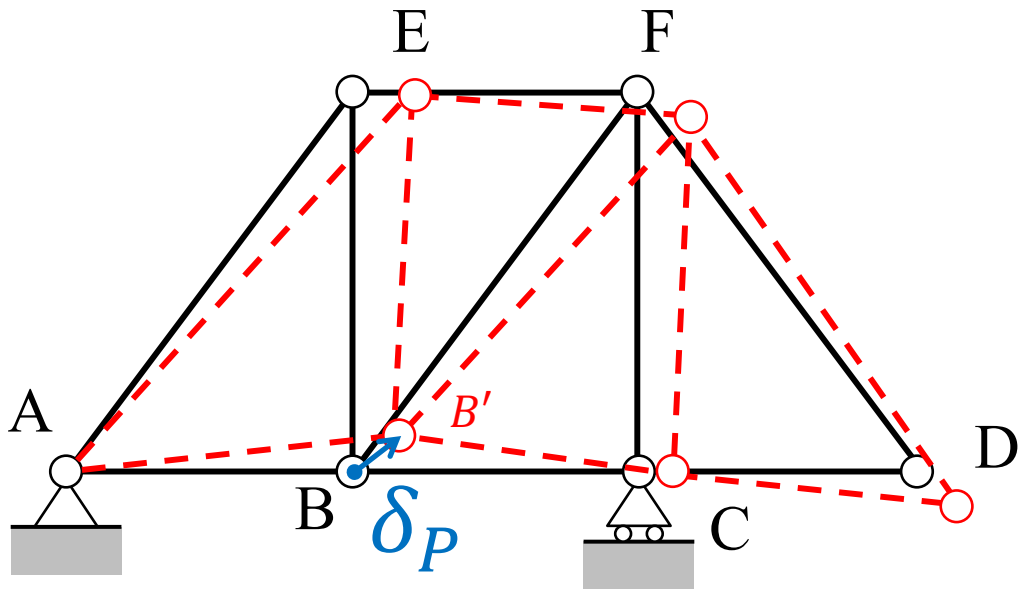
Step 1 –Virtual Analysis



Convenient to set $Q = 1$

1. Apply a unit, dimensionless virtual load **in-line** with the real displacement, δ_P, that we want to find;
2. Perform a truss analysis to find all truss member virtual axial forces, F_{Qi}

Step 2 – Use the Principle of Virtual Work to Find δ_P



A_i = Cross sectional area
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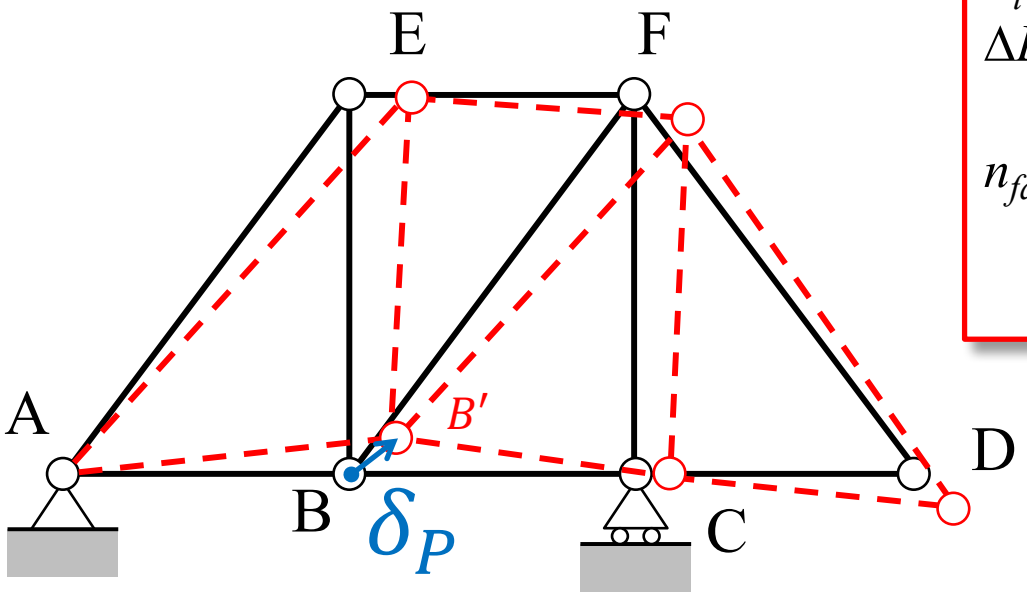
$$1 \cdot \delta_P = \sum_{i=1}^{n_T} F_{Qi} \alpha_i \Delta T_i L_i$$

From Step 1 –virtual analysis

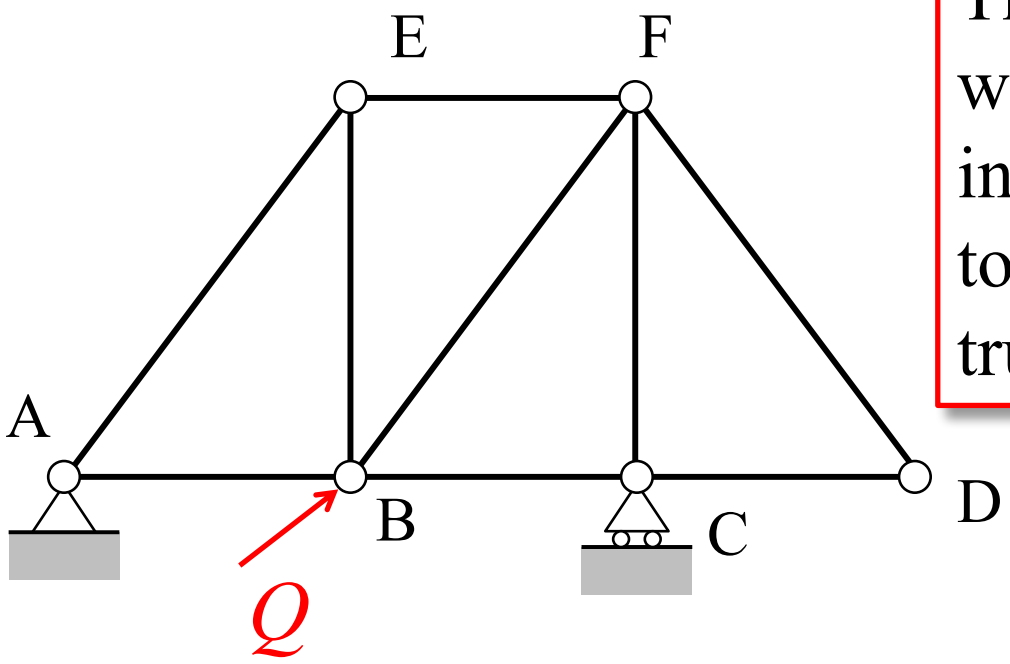
Consider a Truss Structure Subjected To Fabrication Errors to Some or All Members



L_i = Length of truss member
ΔL_{ifabr} = Fabrication error of truss member
n_{fabr} = Total number of truss members subjected to fabrication errors

We want to find the deflection of joint B due to fabrication errors in n_{fabr} truss members

Apply Virtual Force



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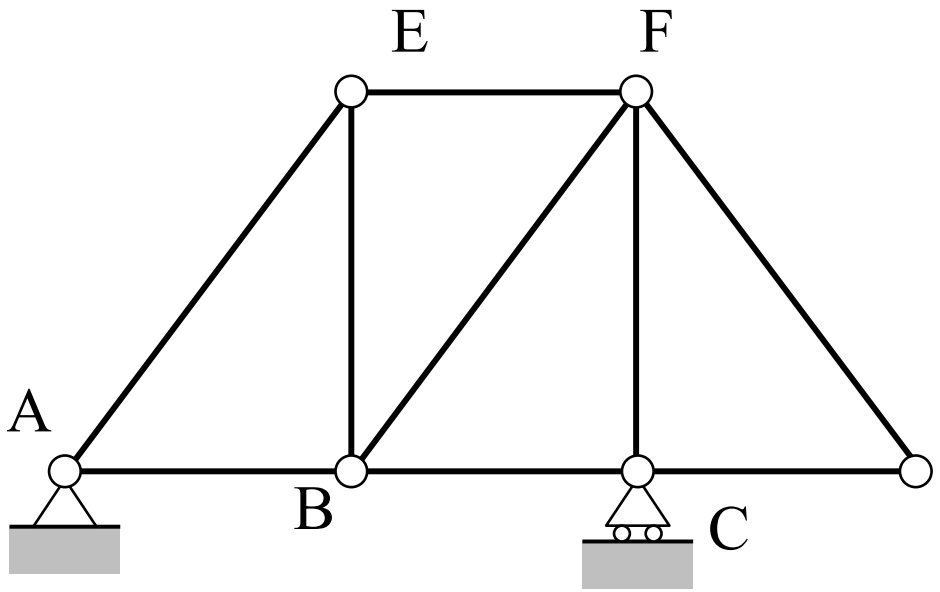

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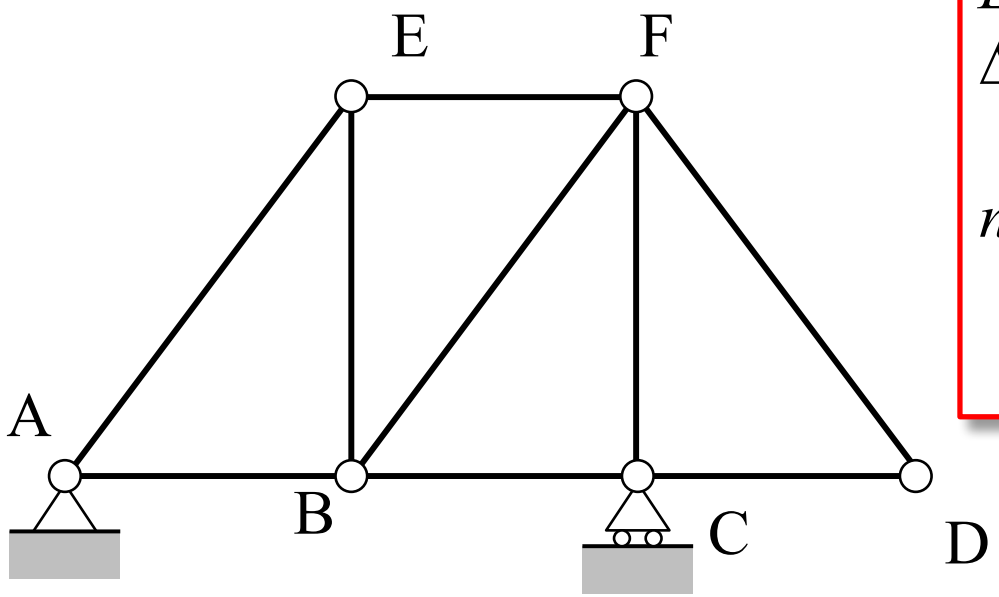
The real fabrication error causes an axial deformation of each truss member, $\Delta L_{Pi} = \Delta L_{ifabr}$

Virtual Strain Energy



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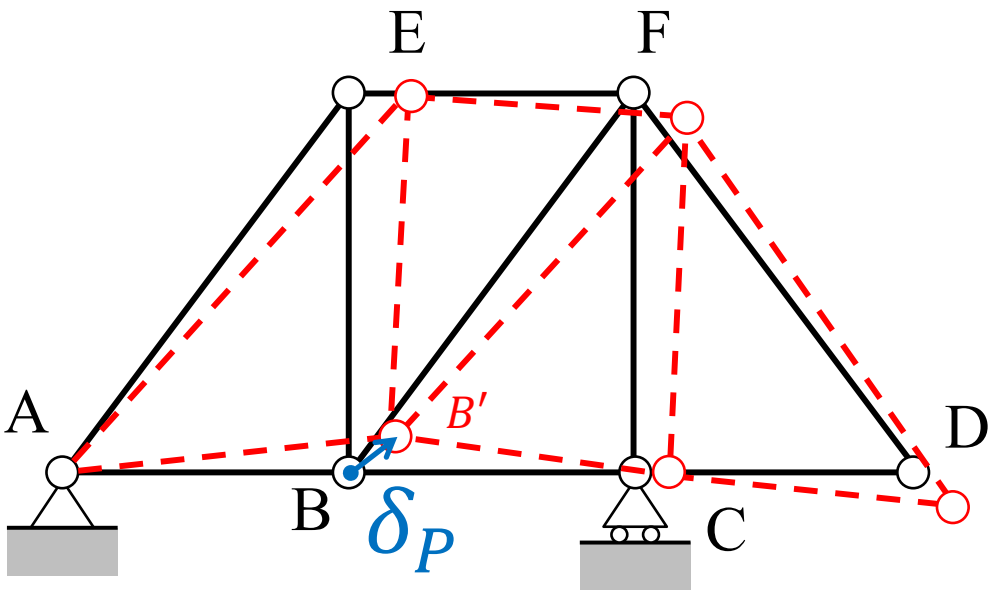

L_i = Length of truss member
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Real Deformation

$$Q\delta_P = \sum_{i=1}^{n_{fabr}} F_{Qi}\Delta L_{ifabr}$$

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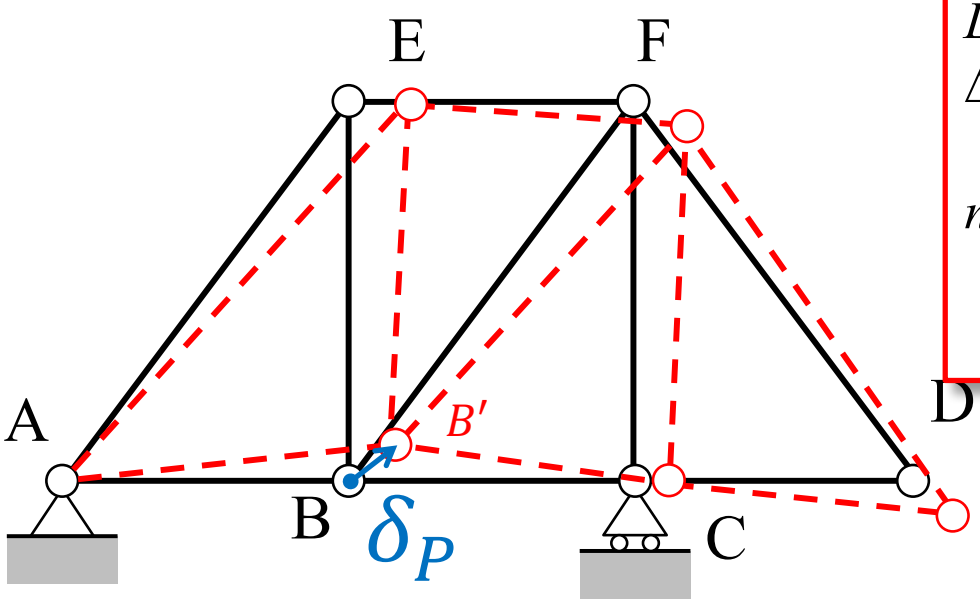

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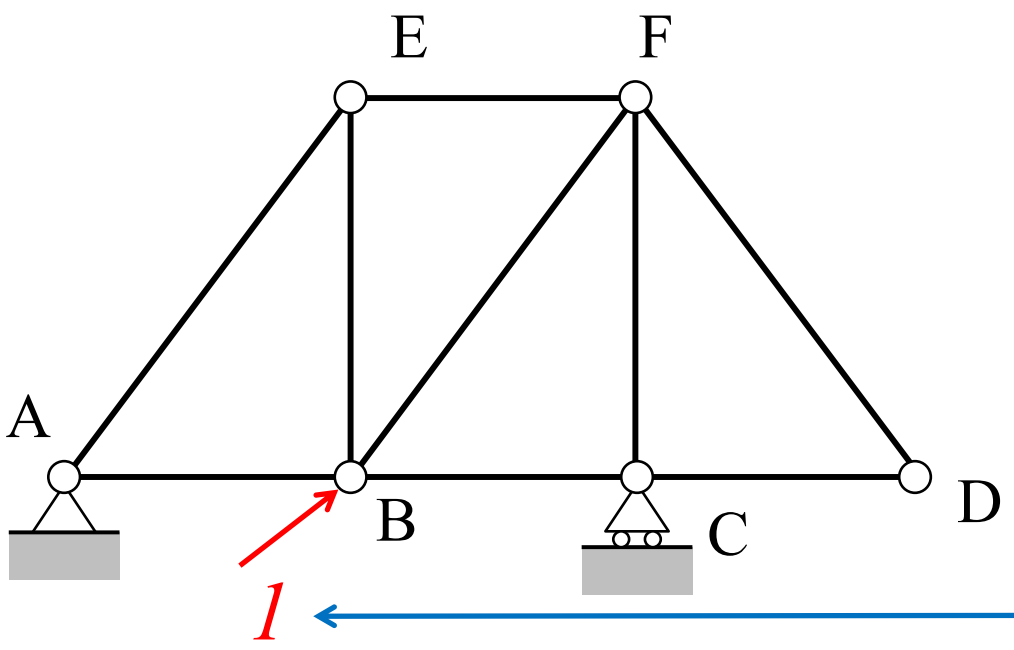

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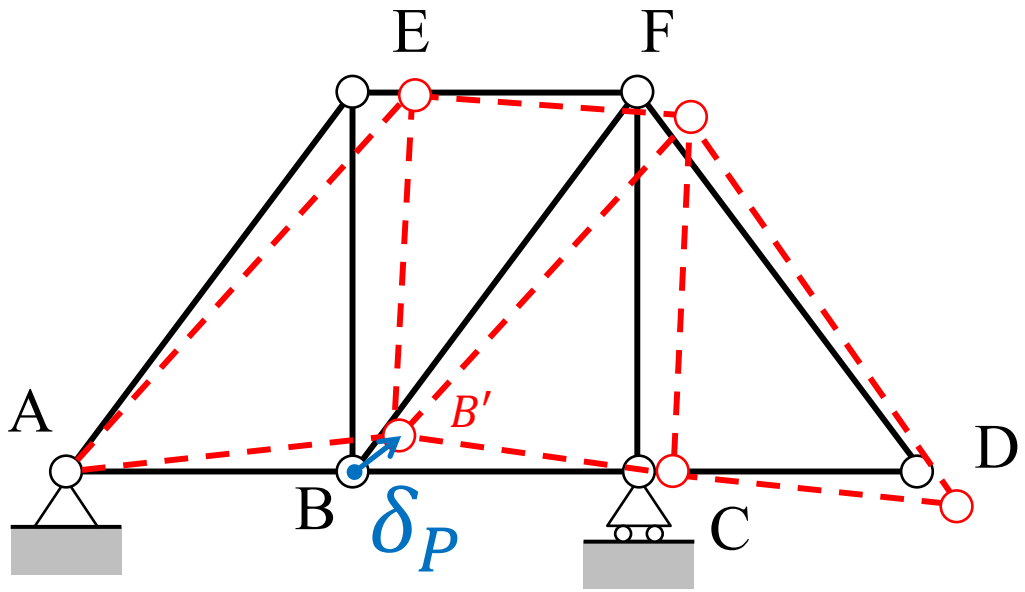

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