Shear Force, Bending Moment, and Axial Force Diagrams for a Frame

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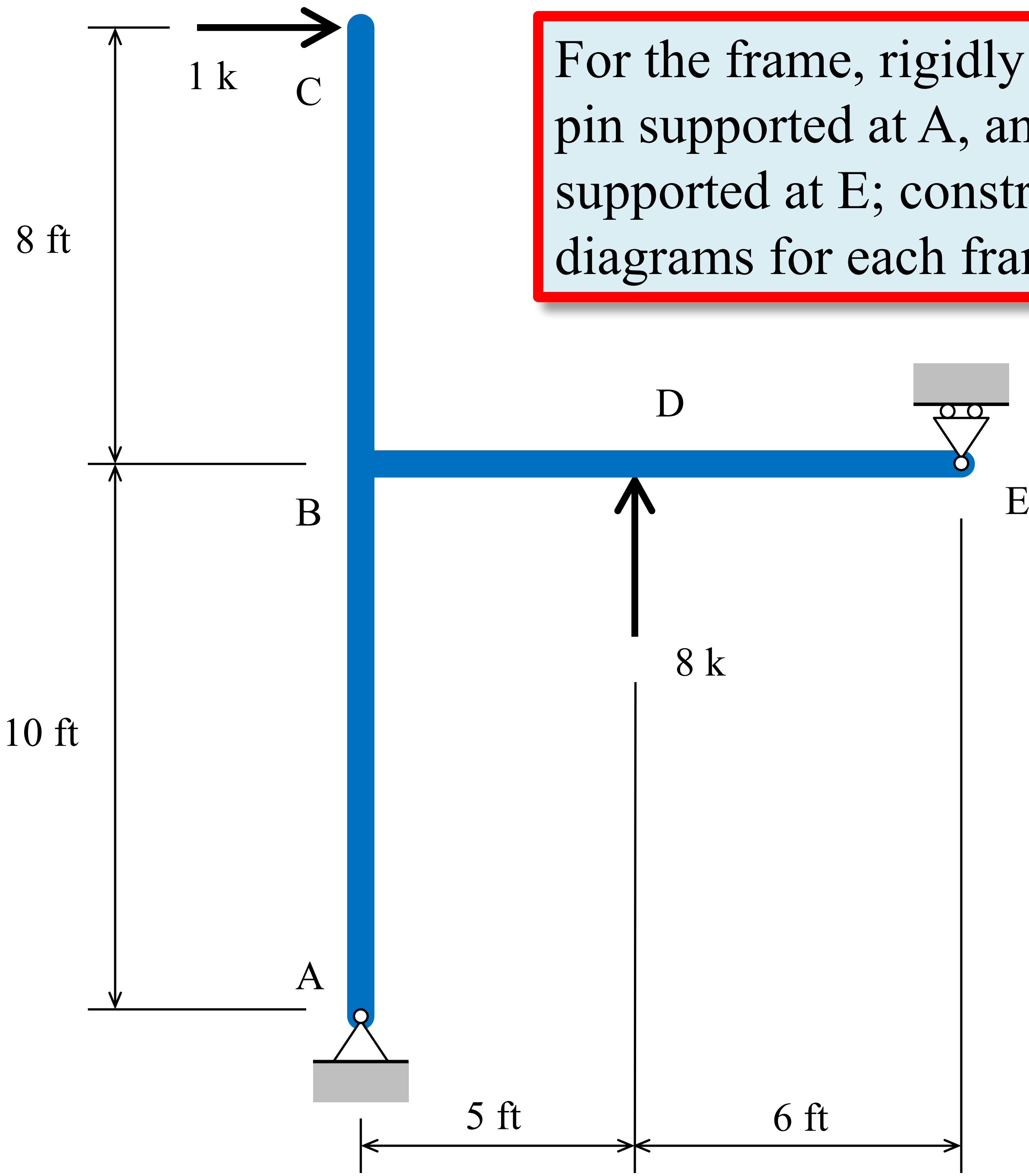
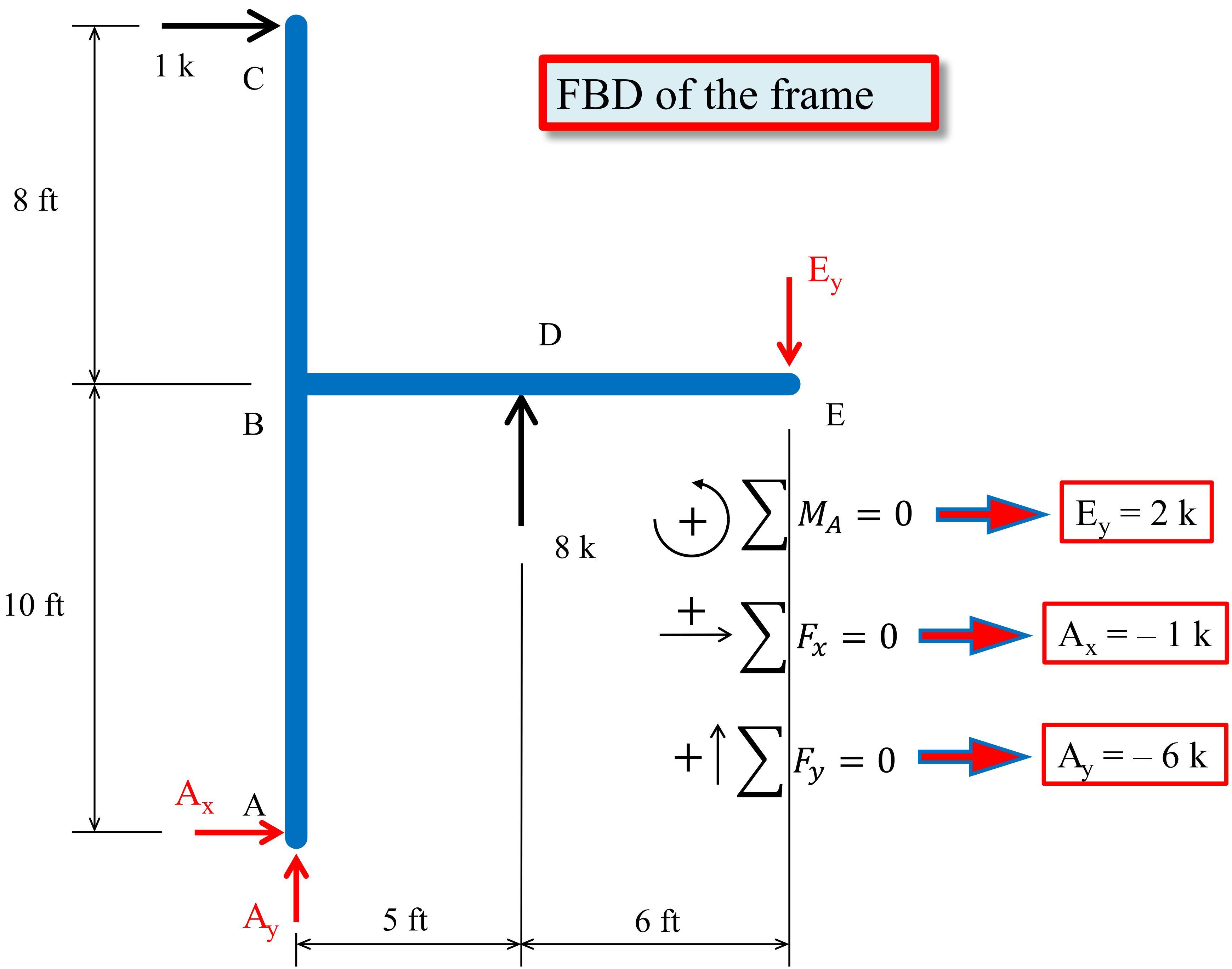
General procedure for the construction of internal force diagrams

1. Find all of the external forces and draw the external force diagram;
2. Choose a sign convention for each diagram;
3. If necessary, choose a reference coordinate system:
4. Use equilibrium analysis or differential and integral relationships to construct internal force functions;
   - Cut structure at appropriate sections,
   - The FBD on either side of the cut may be analyzed,
   - Indicate unknown internal forces consistent with the chosen sign convention,
   - Plot the internal force function for each segment,
5. Check each diagram for errors;
   - Check discontinuities at location of applied forces in shear diagram,
   - Check discontinuities at location of applied moment in moment diagram,
   - Check differential and integral relationships between distributed load, shear, and bending moment.

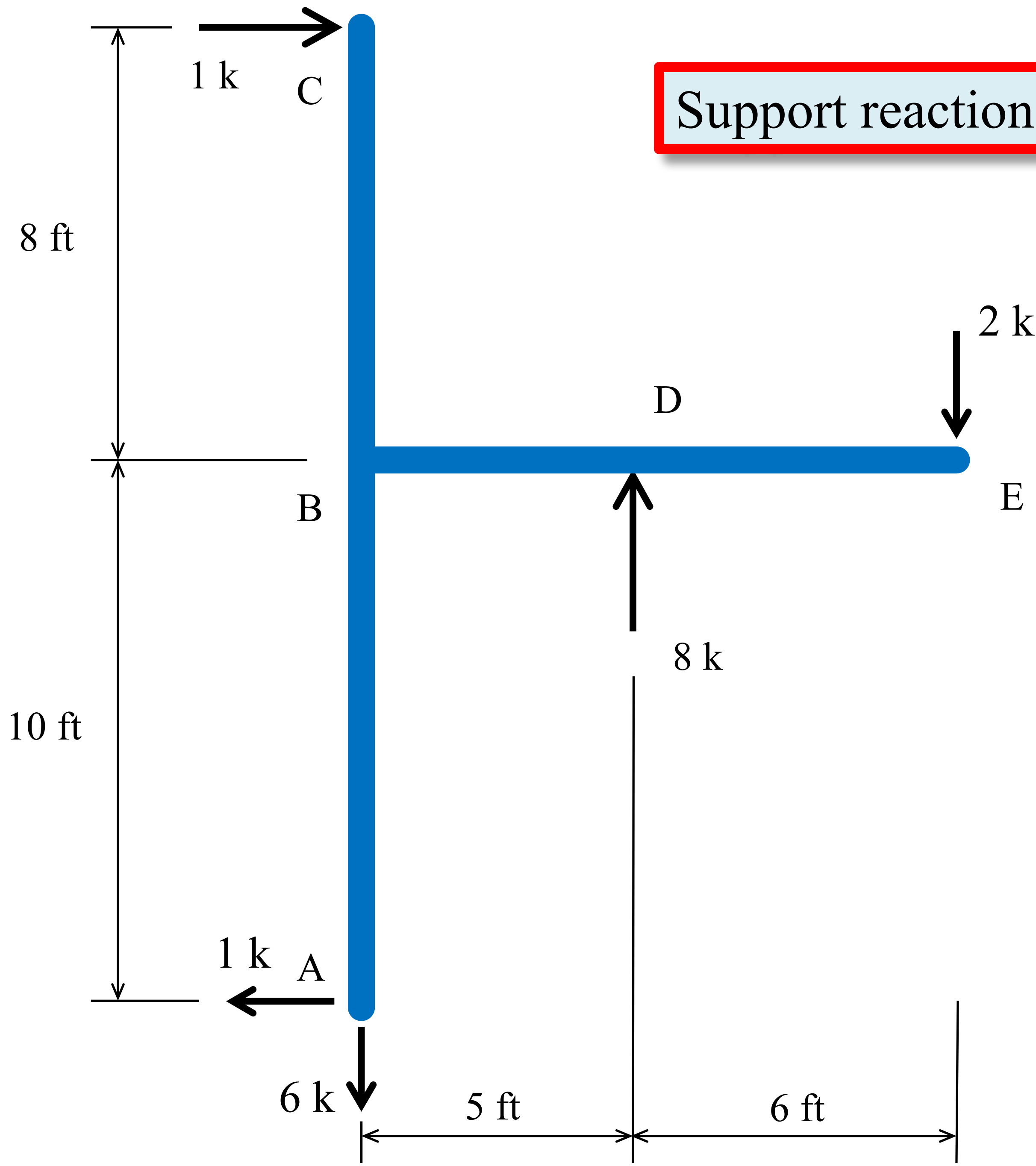


For the frame, rigidly connected at B, pin supported at A, and roller supported at E; construct V, M and F diagrams for each frame member.



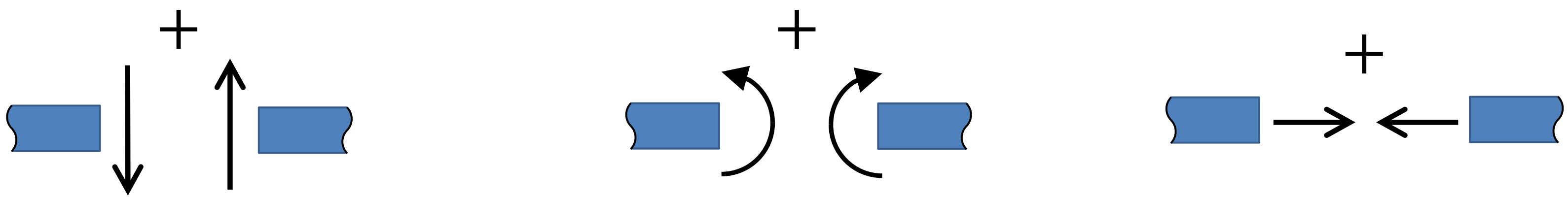
FBD of the frame
1 k
C
8 ft
D
E_y
B
E
10 ft
8 k
A
A_x
A_y
5 ft
6 ft
$\circlearrowleft + \sum M_A = 0$
$E_y = 2$ k
$\xrightarrow{+} \sum F_x = 0$
$A_x = -1$ k
$+\uparrow \sum F_y = 0$
$A_y = -6$ k

Support reactions

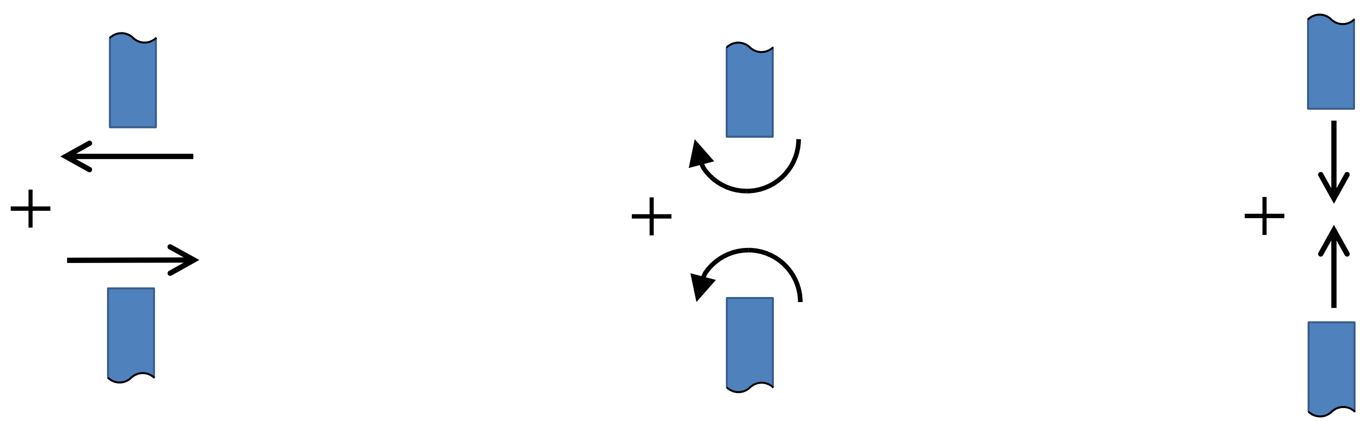


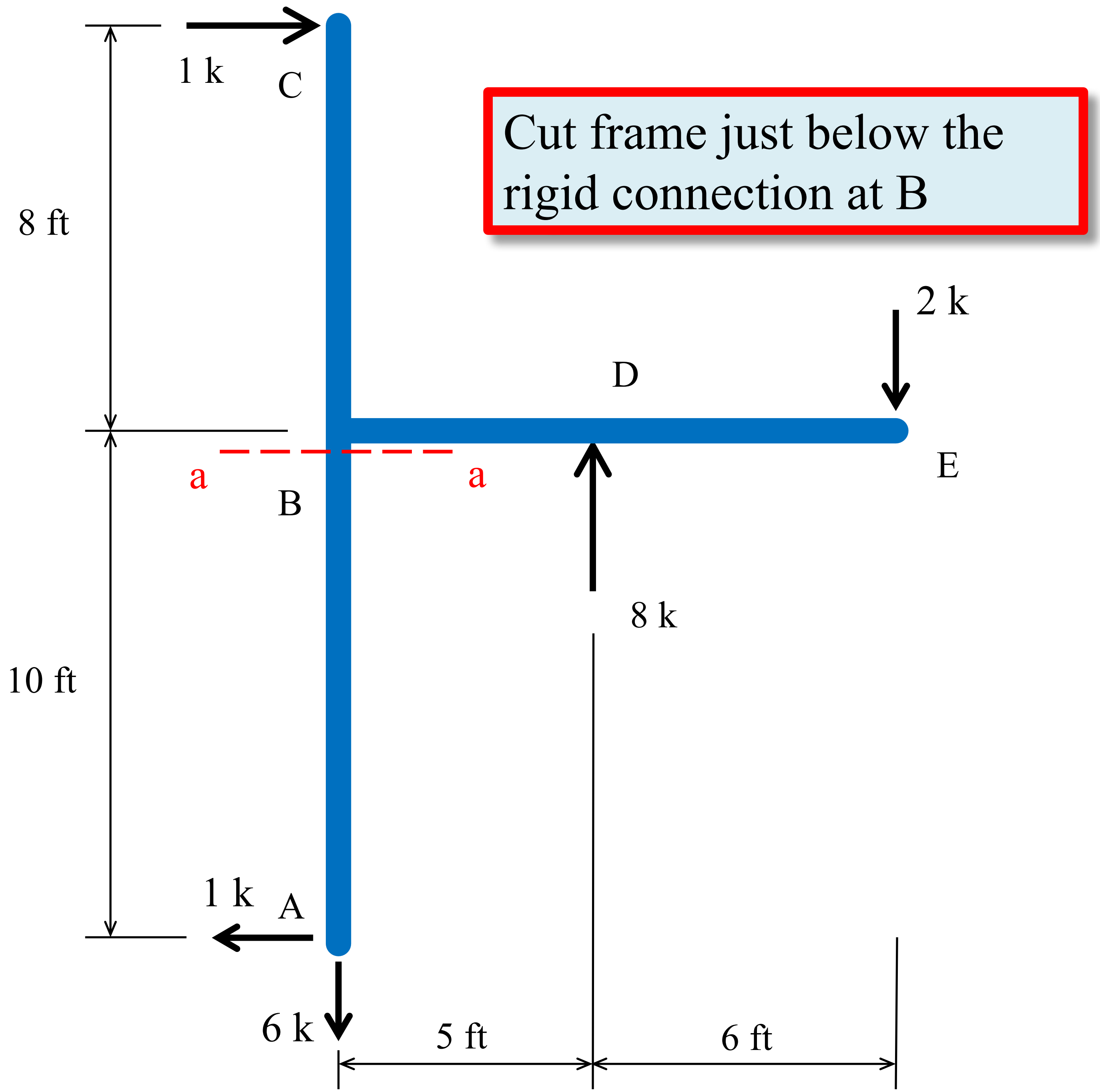
Choose sign convention for internal forces for both horizontal and vertical members

For horizontal member BDE

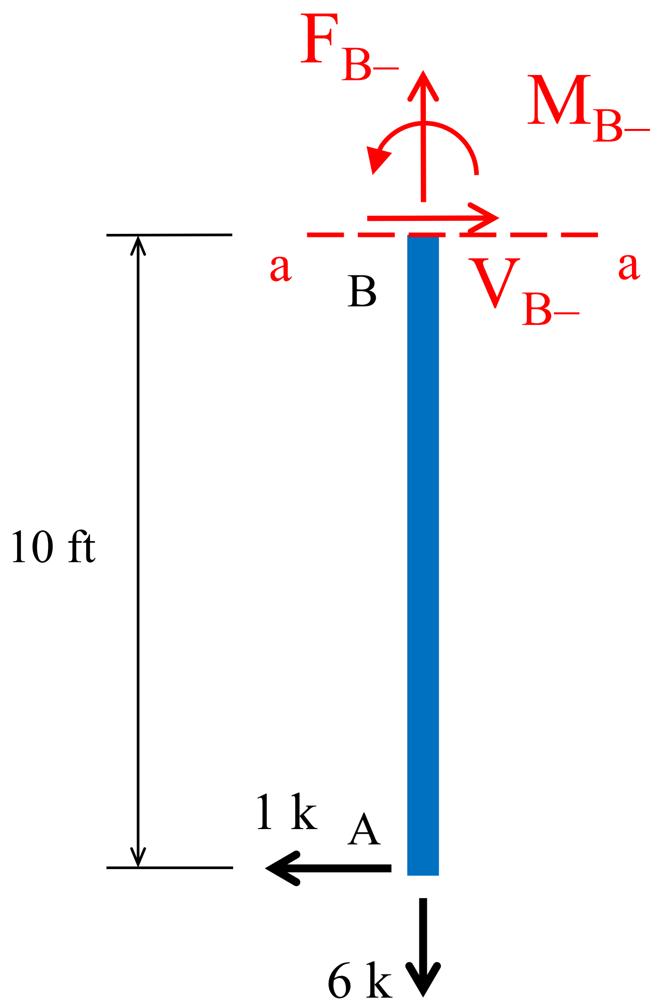


For vertical member ABC




Cut frame just below the rigid connection at B
1 k
C
8 ft
2 k
D
a
a
B
E
8 k
10 ft
1 k
A
6 k
5 ft
6 ft

FBD of segment AB



Unknown internal forces assumed to act in their positive senses

$$\circlearrowleft + \sum M_B = 0$$

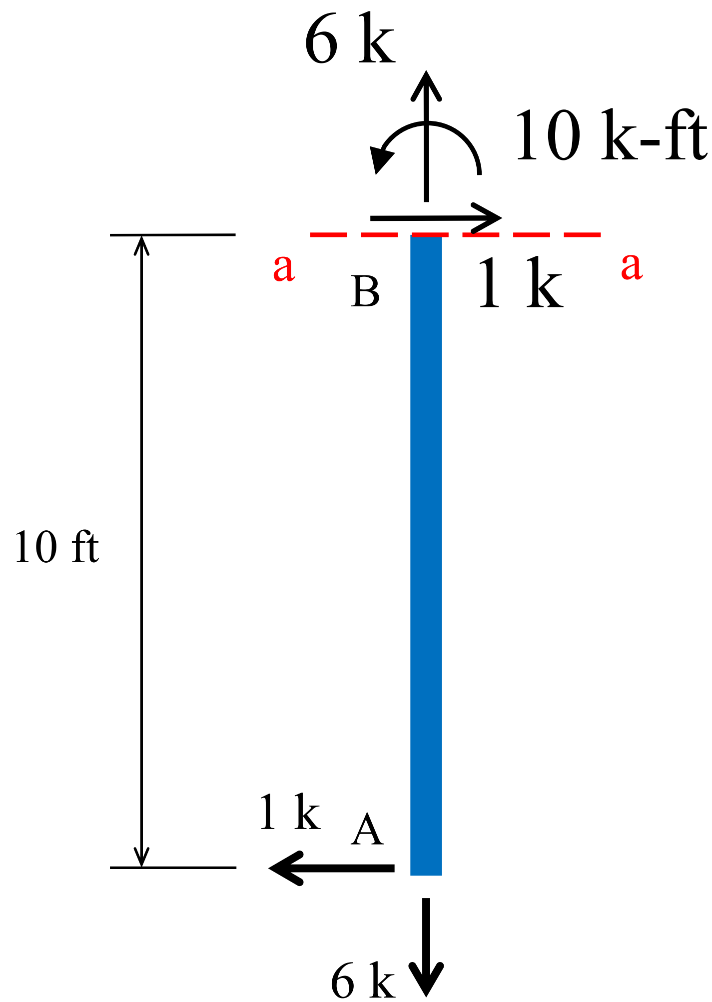
From force equilibrium

$$\xrightarrow{+} \sum F_x = 0 \Rightarrow V_{B-} = 1 \text{ k}$$

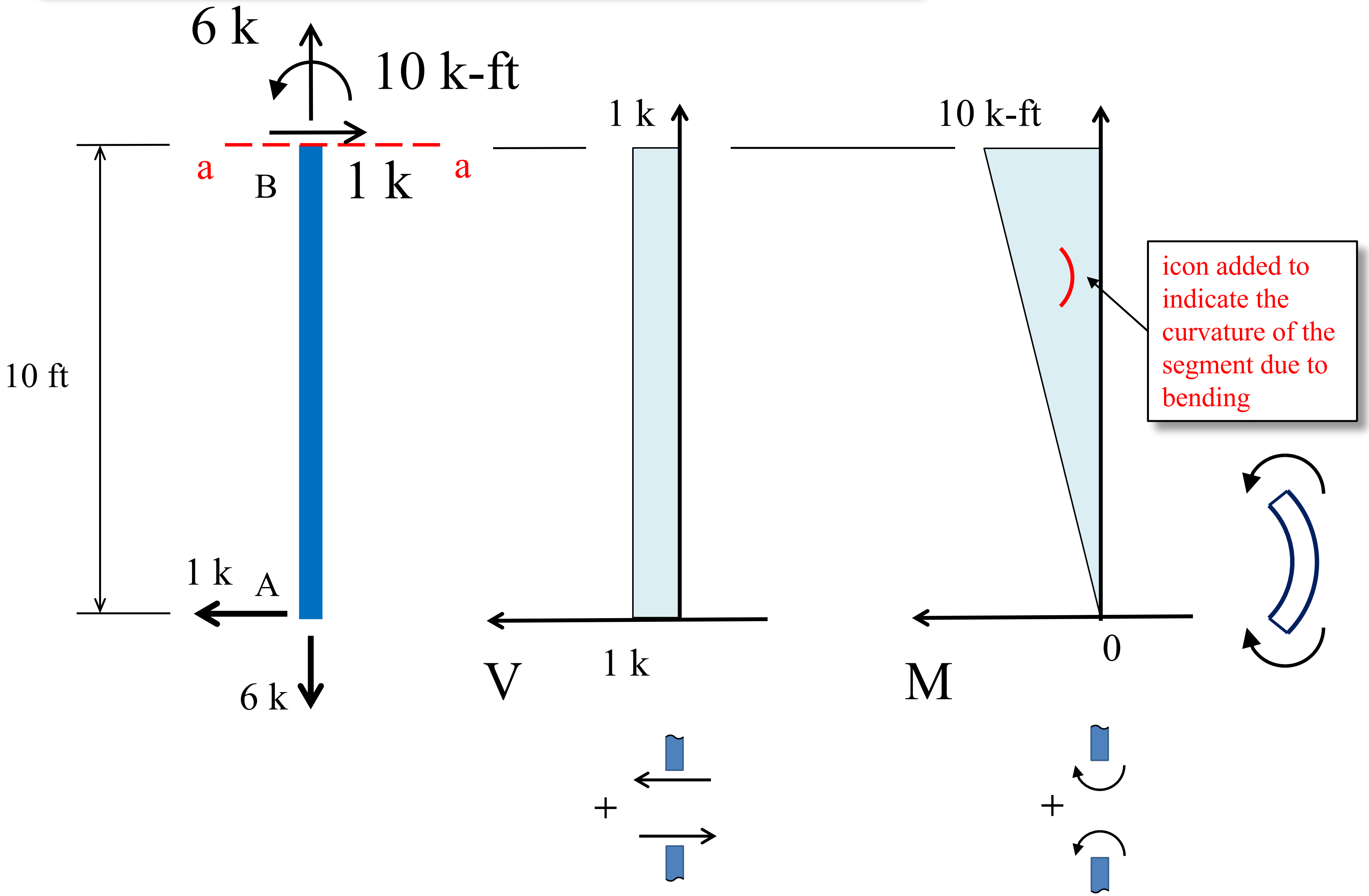
$$+\uparrow \sum F_y = 0 \Rightarrow F_{B-} = 6 \text{ k}$$

$M_{B-} = 10$ k-ft

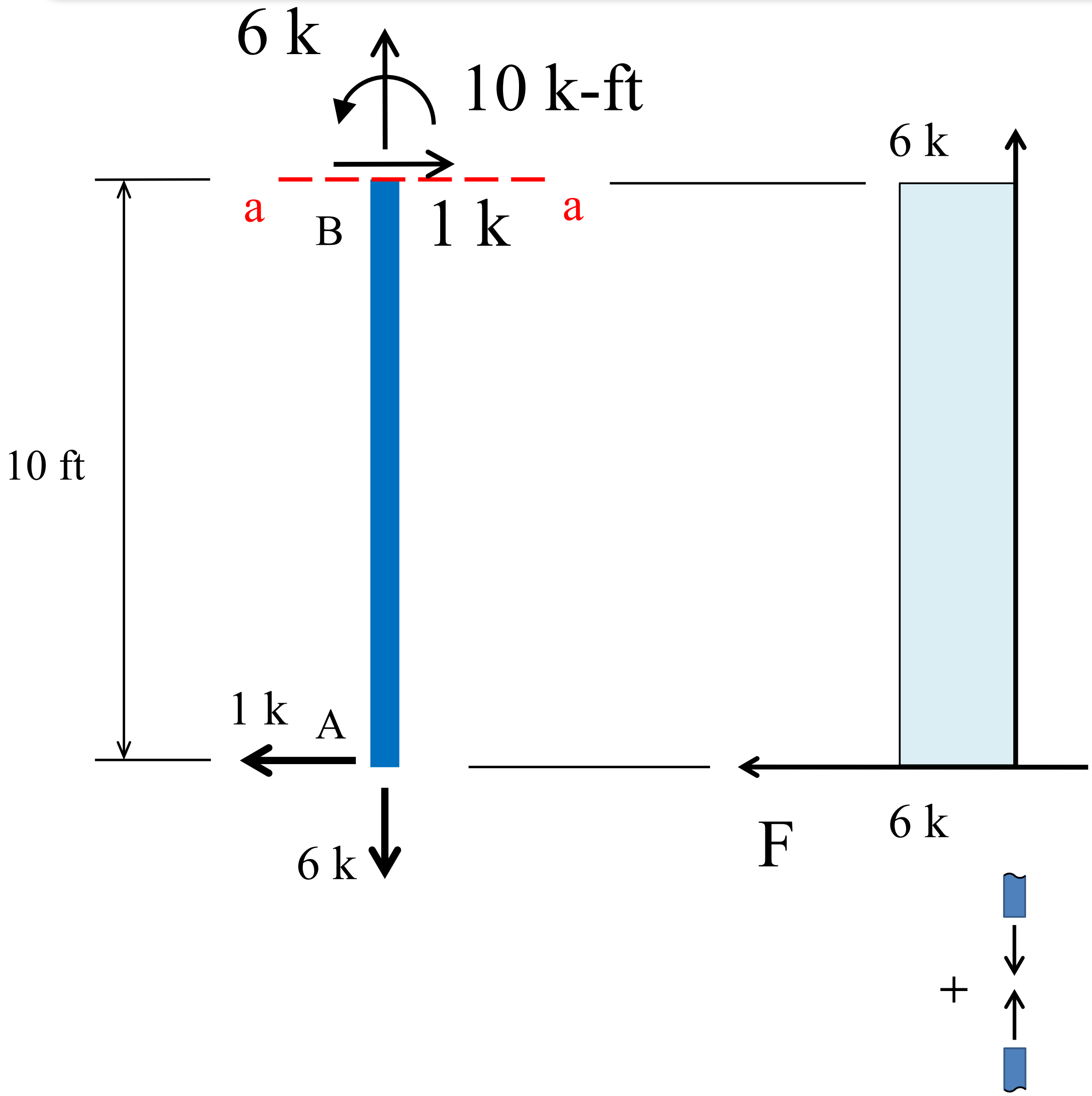
FBD of segment AB showing internal forces just below B

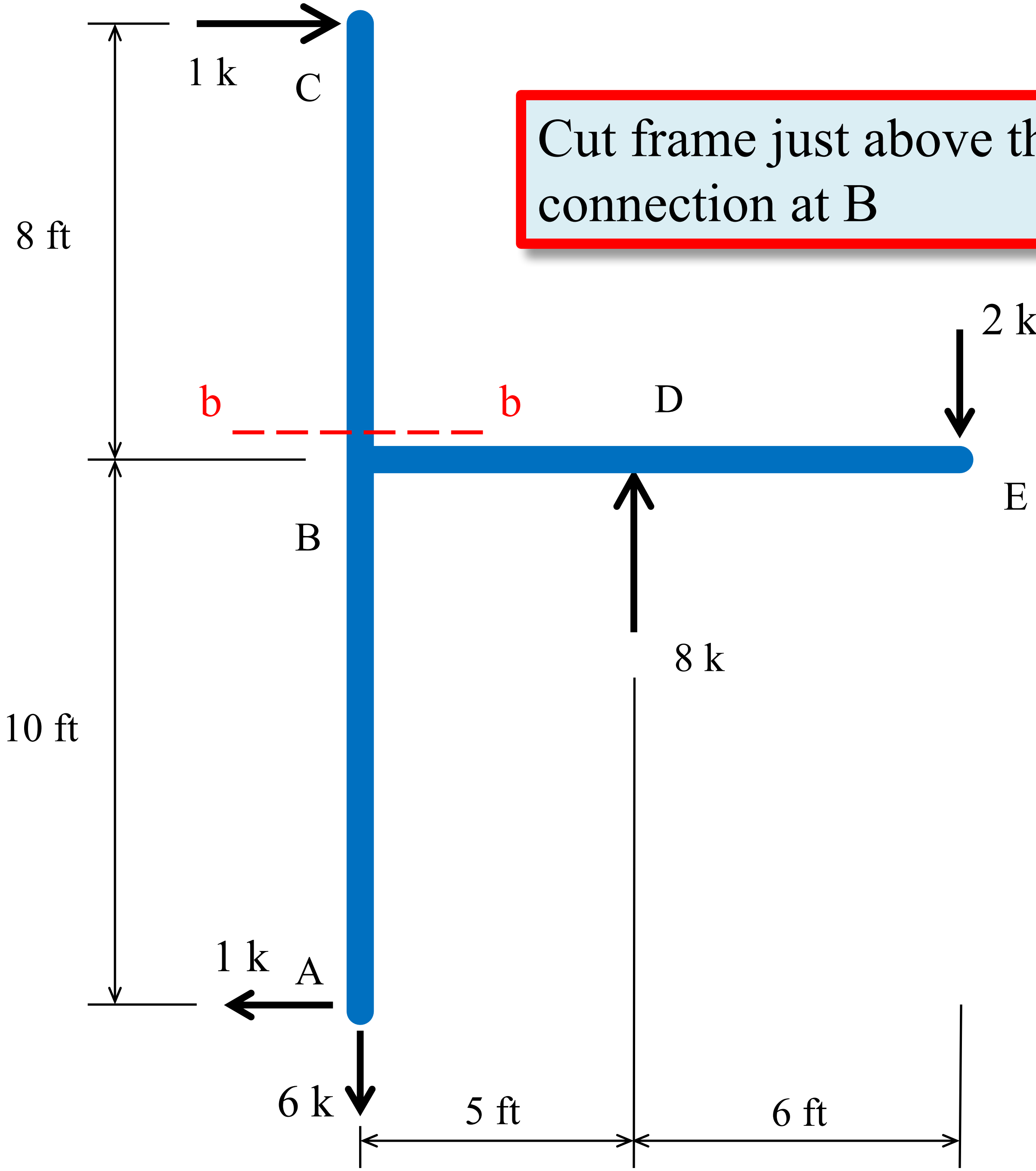


Plot V and M diagrams for segment AB



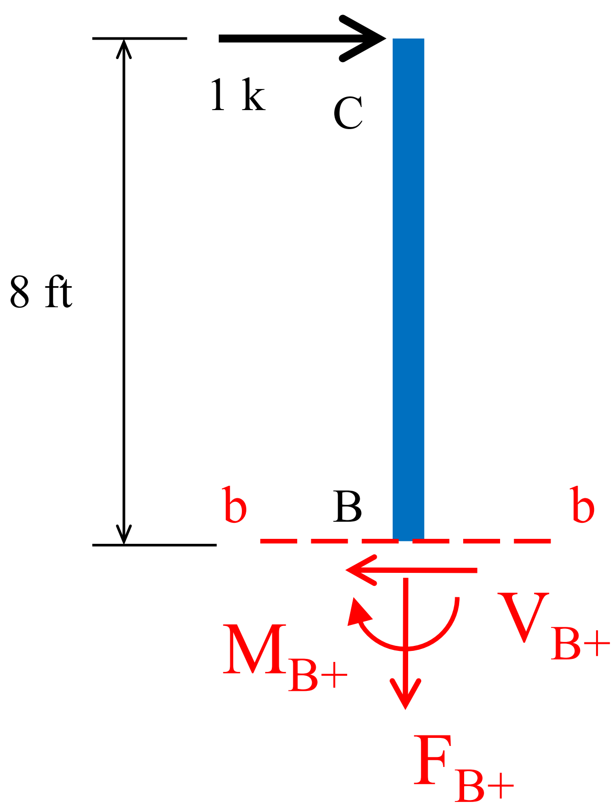
Plot axial force diagram for segment AB





Cut frame just above the rigid connection at B

FBD of segment BC



Unknown internal forces assumed to act in their positive senses

$$\circlearrowleft + \sum M_B = 0$$

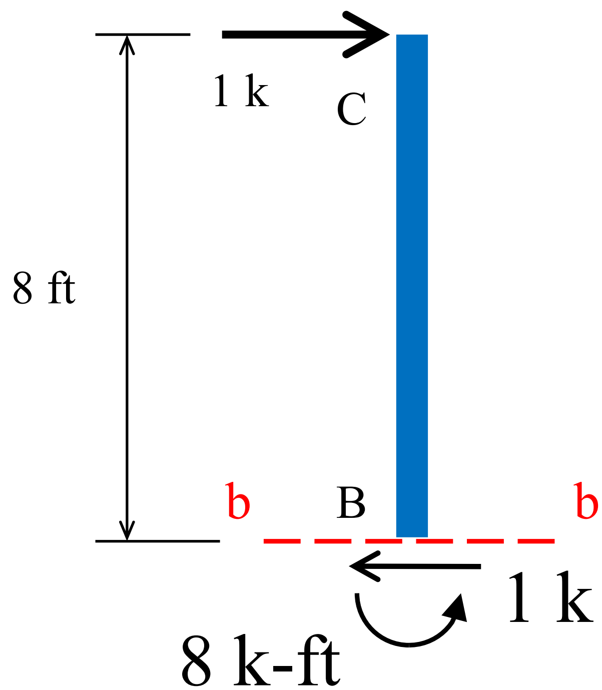
From force equilibrium

$$\xrightarrow{+} \sum F_x = 0 \Rightarrow V_{B+} = 1 \text{ k}$$

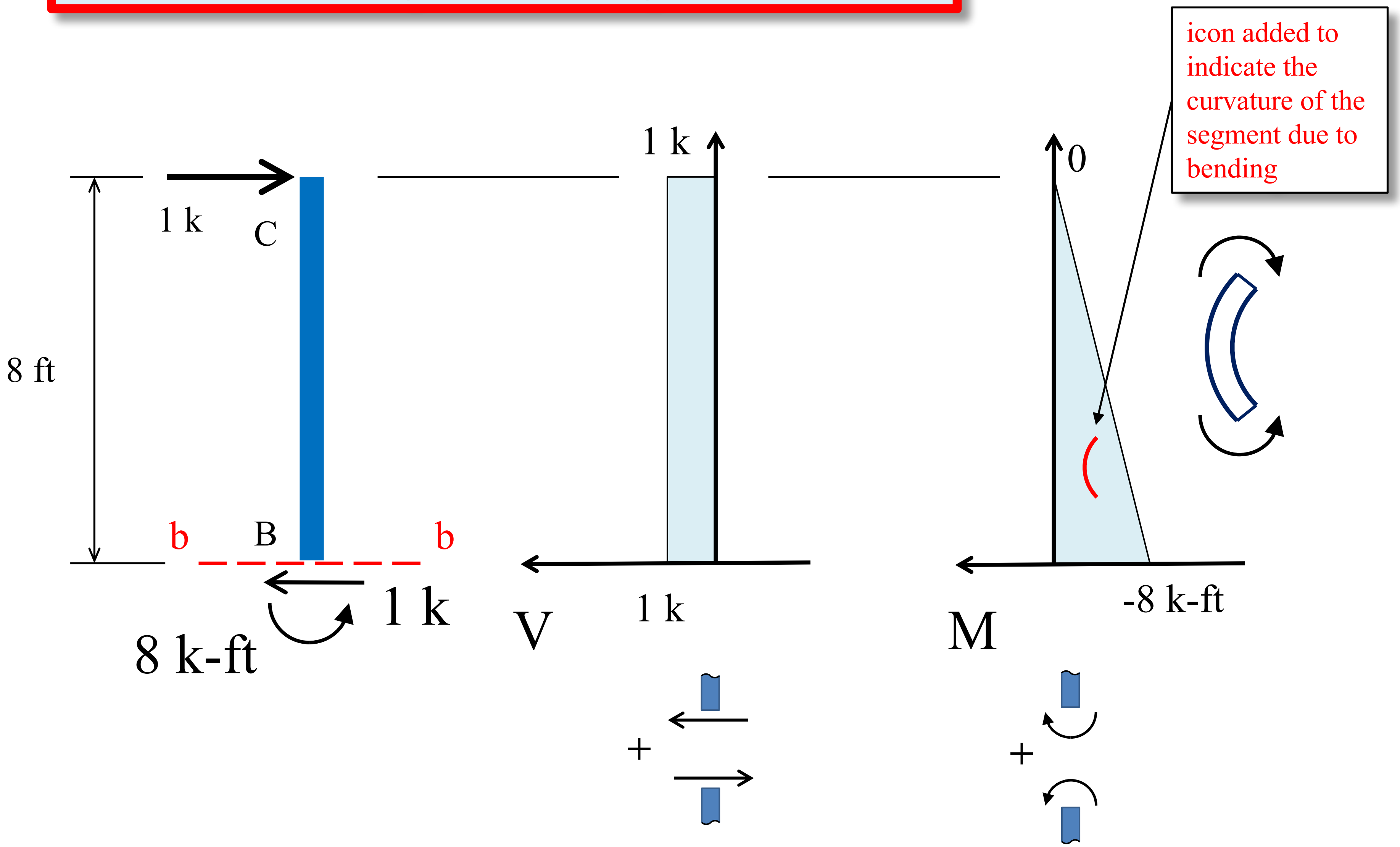
$$+\uparrow \sum F_y = 0 \Rightarrow F_{B+} = 0$$

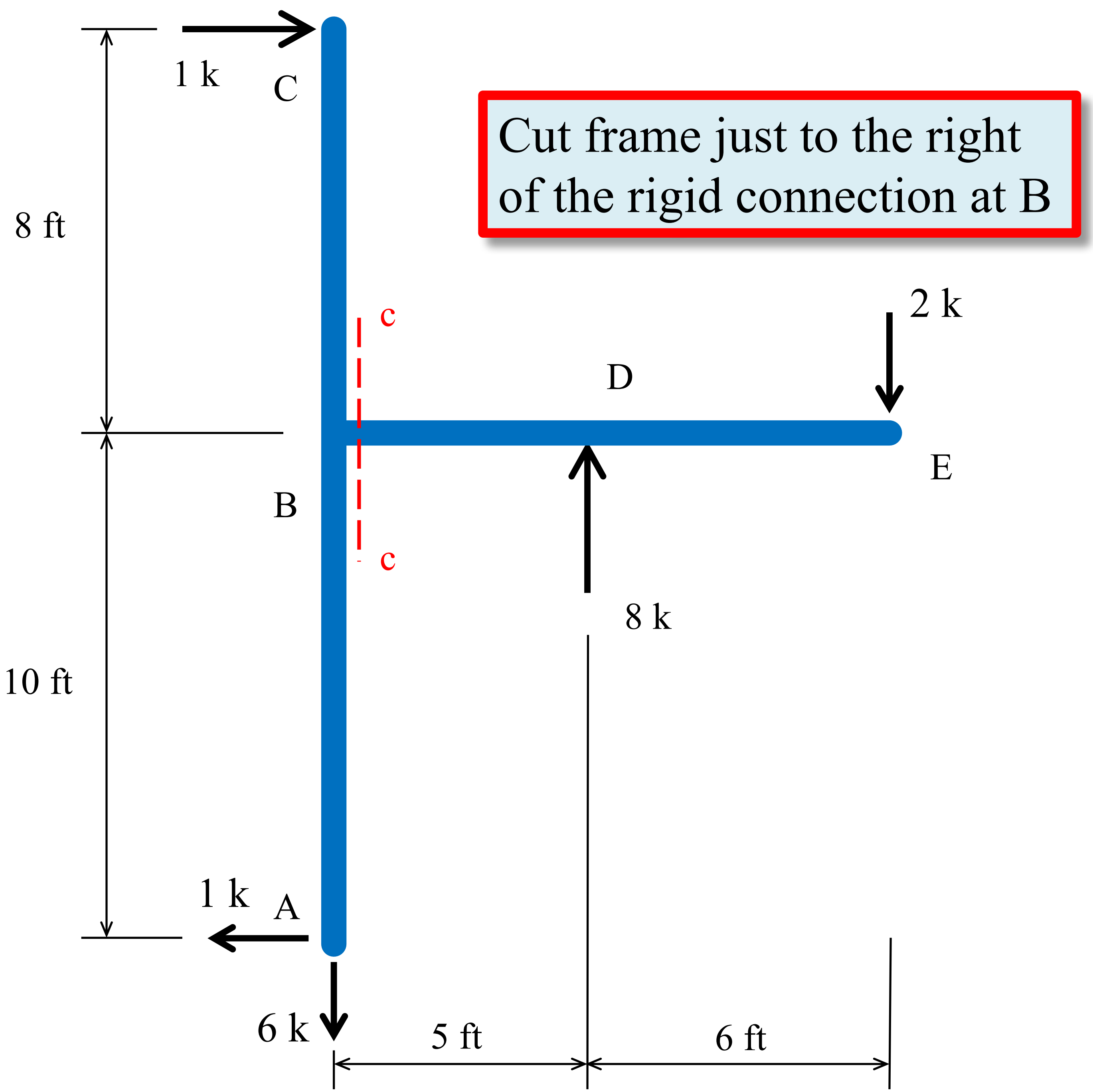
$M_{B+} = -8$ k-ft

FBD of segment BC showing internal forces just above B



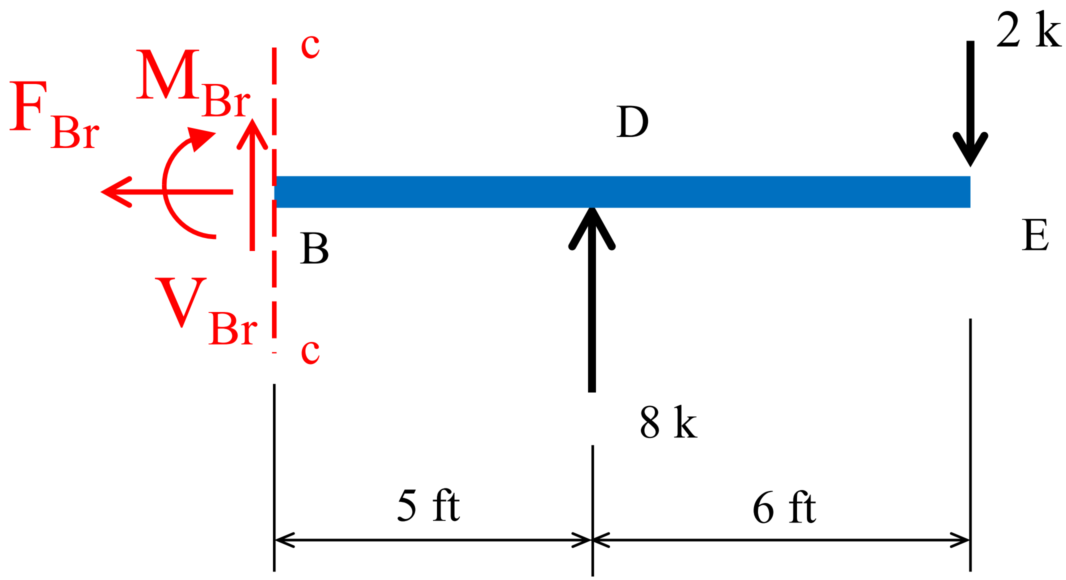
Plot V and M diagrams for segment BC




Cut frame just to the right
of the rigid connection at B
1 k
C
8 ft
c
2 k
D
B
E
c
8 k
10 ft
1 k
A
6 k
5 ft
6 ft

FBD of segment BDE

Unknown internal forces assumed to act in their positive senses



$$\circlearrowleft + \sum M_B = 0$$

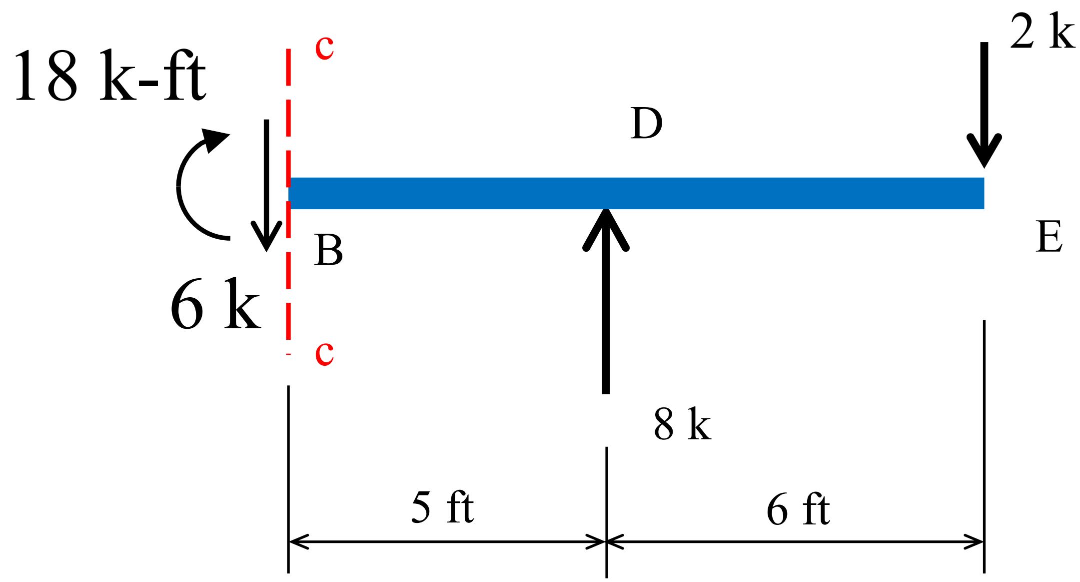
From force equilibrium

$$\xrightarrow{+} \sum F_x = 0 \quad \Rightarrow \quad V_{Br} = -6 \text{ k}$$

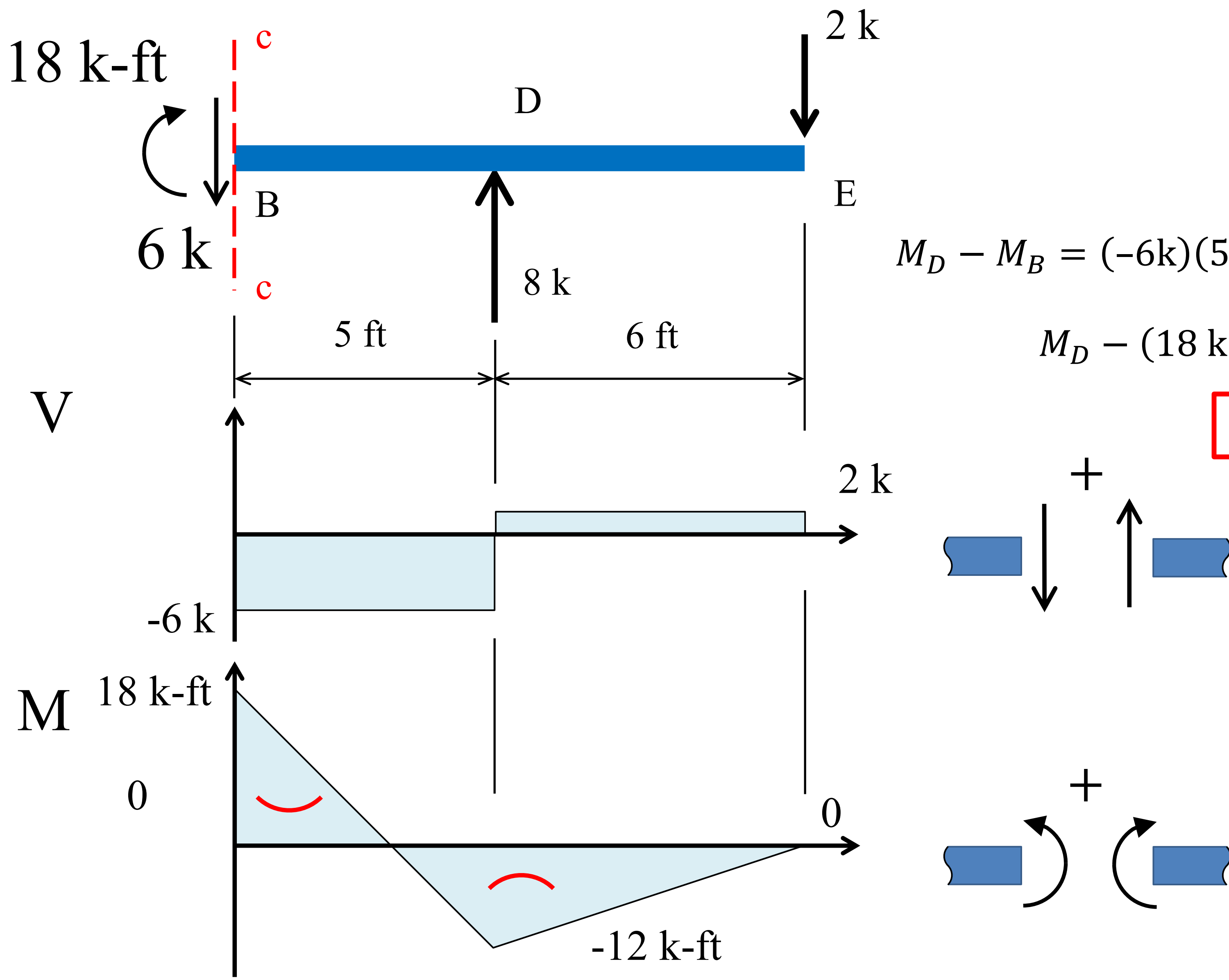
$$+\uparrow \sum F_y = 0 \quad \Rightarrow \quad F_{Br} = 0$$

$M_{Br} = 18$ k-ft

FBD of segment BDE showing internal forces just to the right of B



Plot V and M diagrams for segment BDE



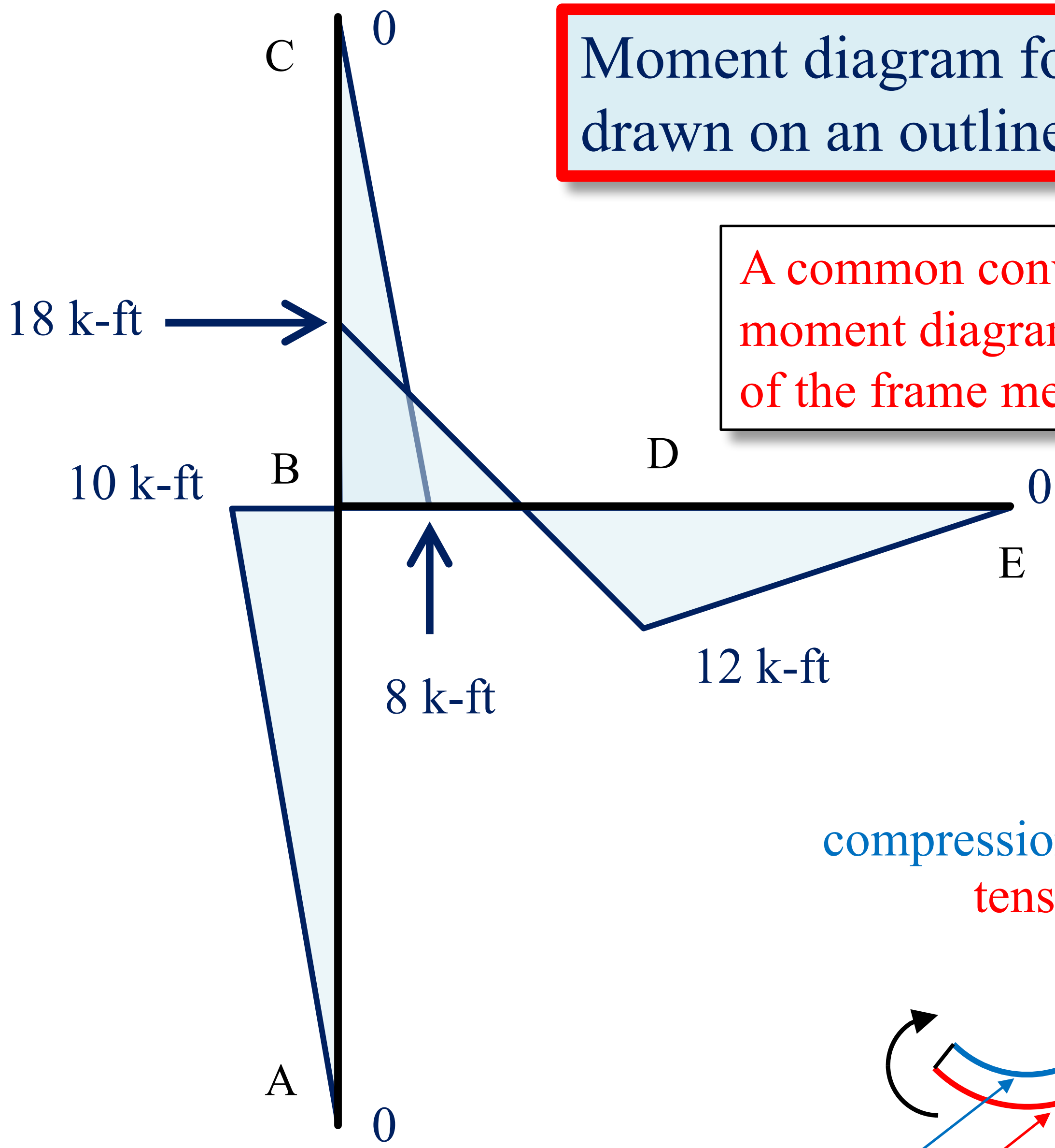
$$M_D - M_B = (-6\text{k})(5\text{ ft}) = -30\text{ k-ft}$$

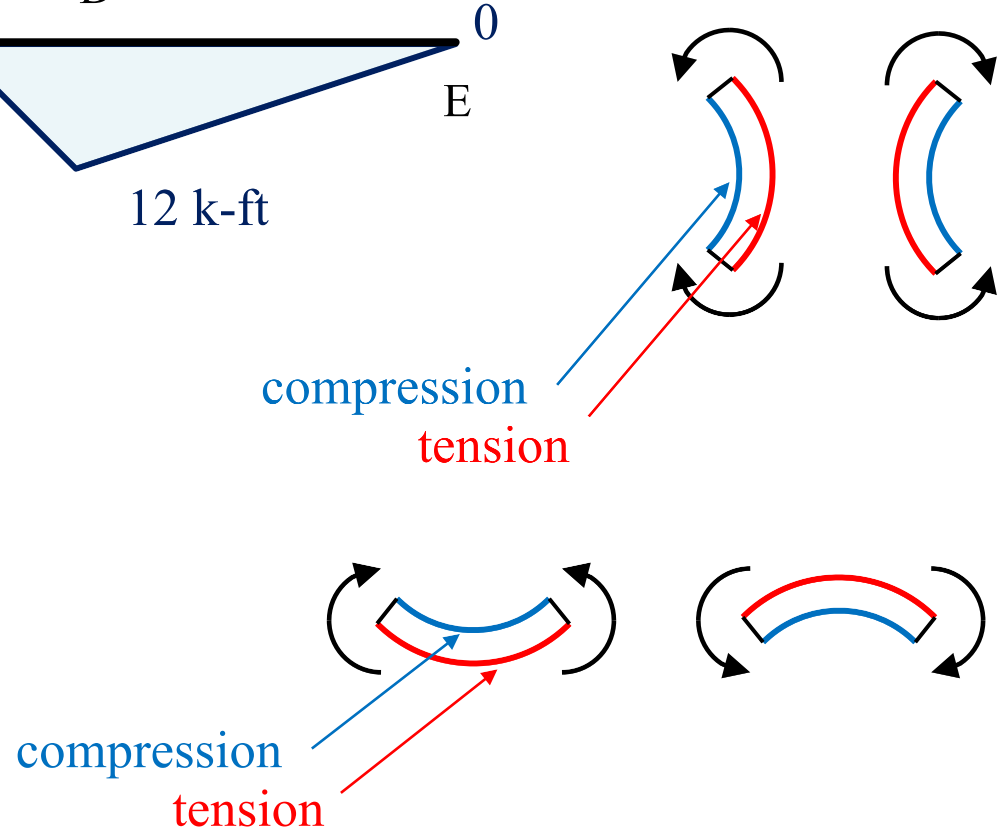
$$M_D - (18\text{ k-ft}) = -30\text{ k-ft}$$

$$M_D = -12\text{ k-ft}$$

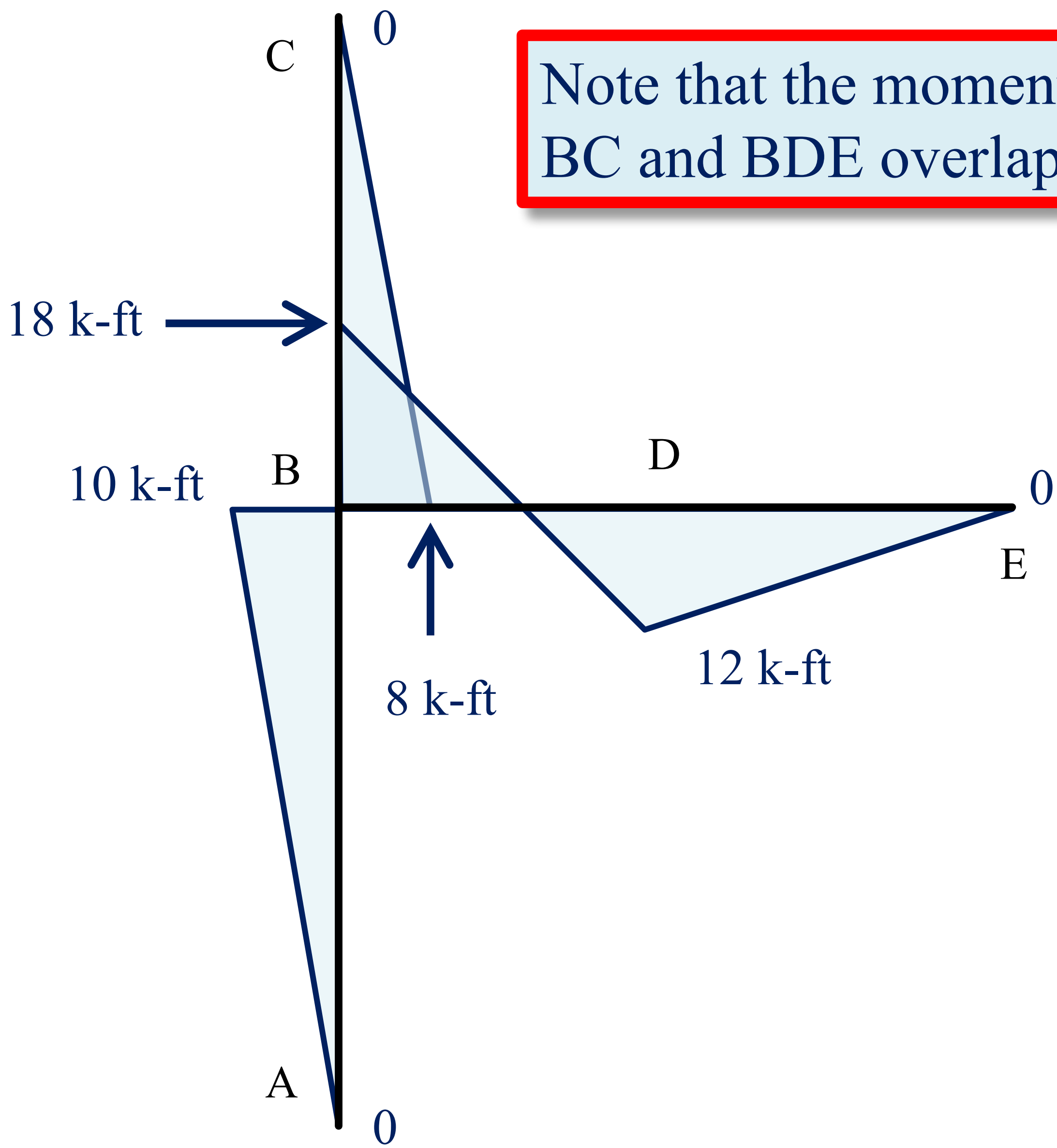
Moment diagram for the entire frame drawn on an outline of the frame

A common convention is to draw the moment diagram on the **compression** side of the frame members

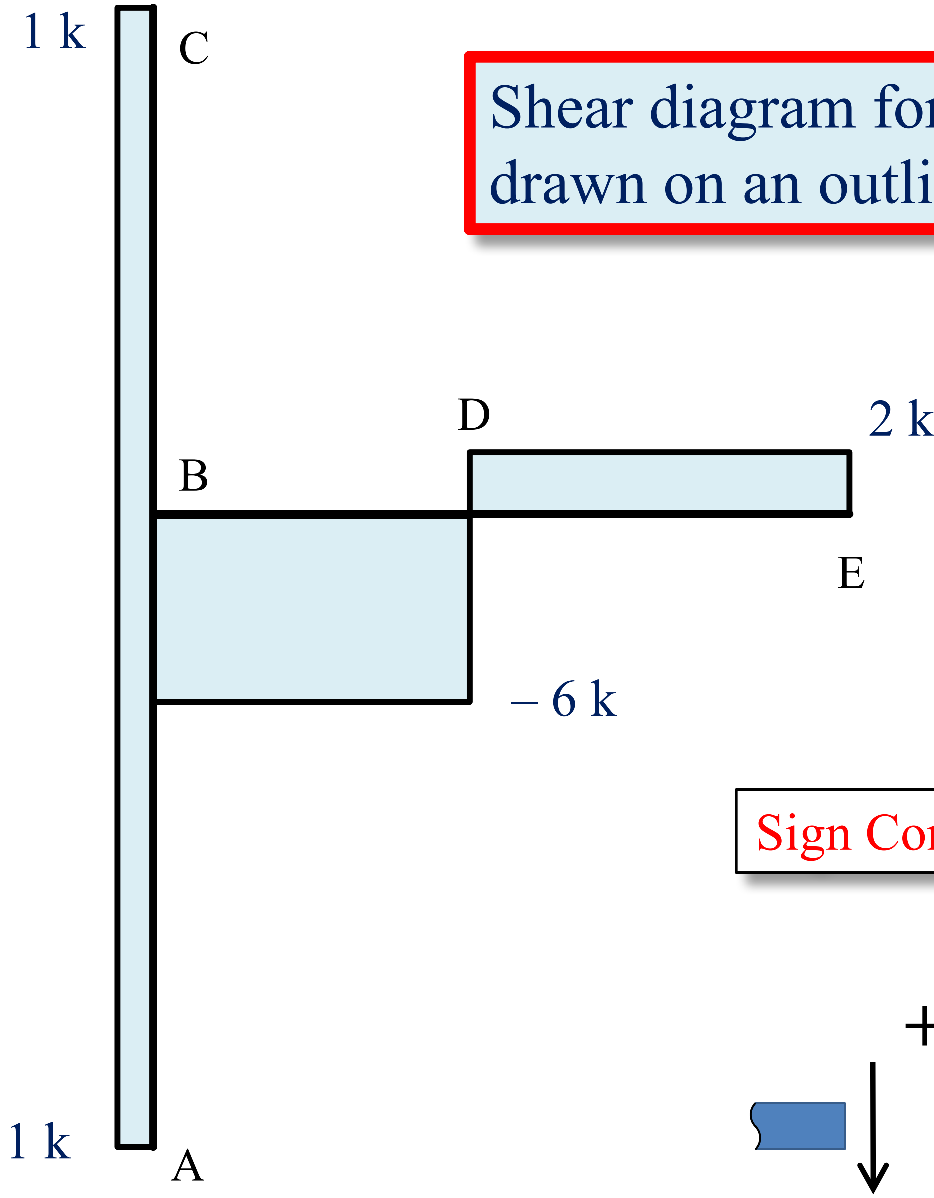




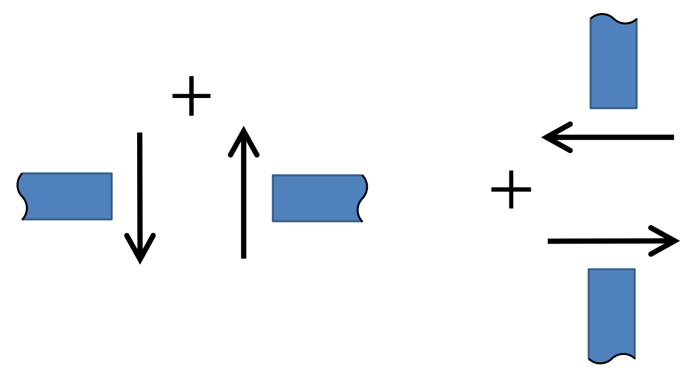
Note that the moment diagrams for segments BC and BDE overlap at point B



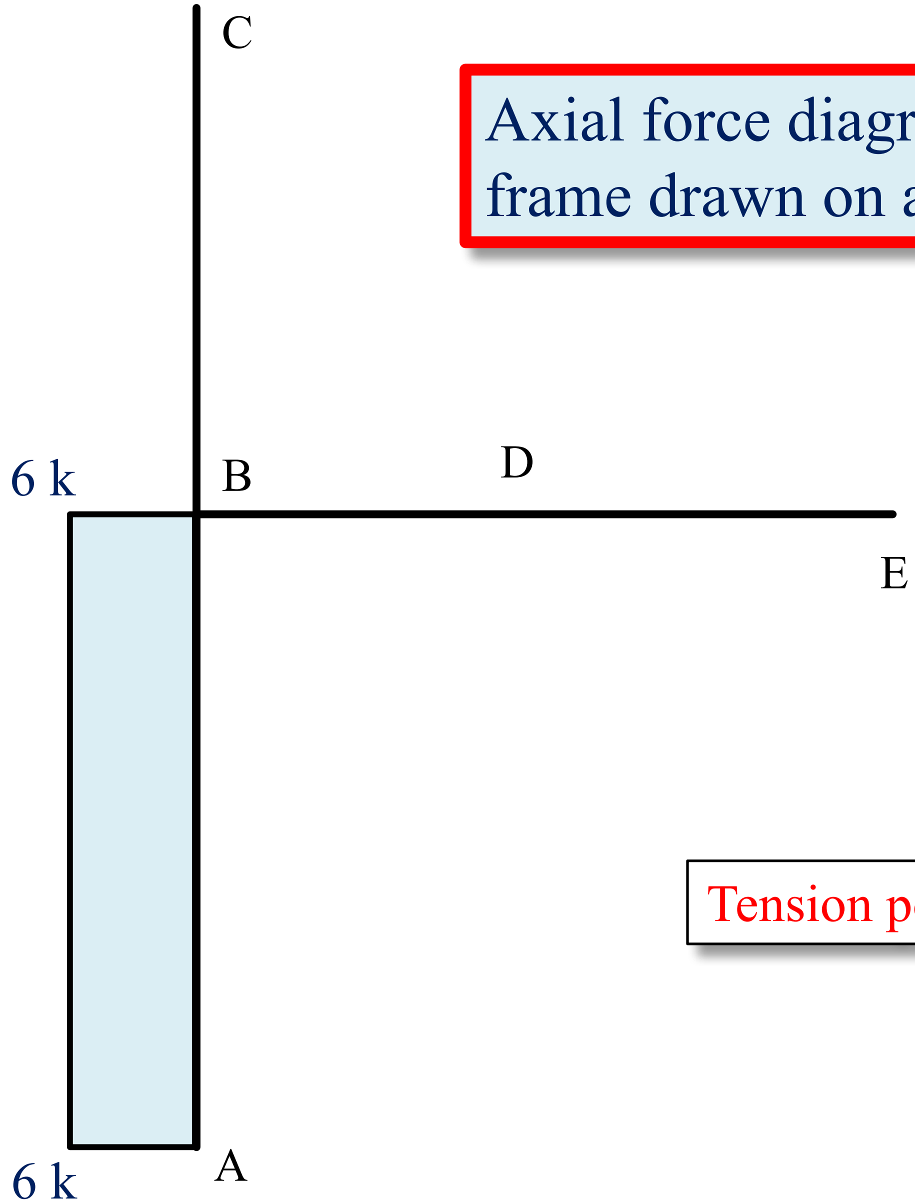
Shear diagram for the entire frame drawn on an outline of the frame



Sign Convention for Shear

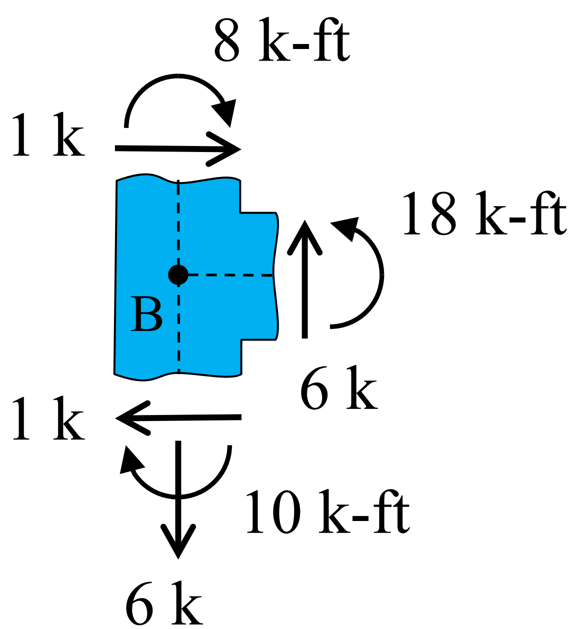


Axial force diagram for the entire frame drawn on an outline of the frame



Tension positive

FBD of Joint B – Joint B is in Equilibrium



$$+\uparrow \sum F_y = 0$$

$$\xrightarrow{+} \sum F_x = 0$$

$$\circlearrowleft + \sum M_B = 0$$