Stability for Planar Structures

Steven Vukazich
San Jose State University

Definition of Determinacy of a Planar Structure

A **statically determinate** structure is one that is stable and all unknown reactive forces can be determined from the equations of equilibrium alone.

A **statically indeterminate** structure is one that is stable but contains more unknown forces than available equations of equilibrium.

We will see later in the course that statically indeterminate structures can be solved but require information on the deformation of the structure.

Definition of Stability of a Structure

A stable structure can support any general loading without the entire structure or any component part of the structure moving as a rigid body. The equations of equilibrium provide the theoretical basis for the assessment of the stability of a structure.

There are two conditions that cause instability:

1. **Partial Constraints;**
2. **Improper Constraints.**

Assessing the Stability of a Structure

The assessment of the stability of a structure should follow the following steps:

1. The entire structure must be cut at all supports (e.g. pins, rollers, fixed supports);
2. A free-body diagram of the entire structure must be drawn and assessed for stability;
3. The structure then must be cut at all locations of known internal force (e.g. internal hinges);
4. Free-body diagrams of each component ”piece” of the structure must be drawn and assessed for stability.

If any of the assessments in Steps 2 or 4 yields an instability, then the structure is deemed unstable.

Assessing Instability Due to Partial Constraints for a Planar Structure

Let:

n = total number of "pieces" associated with the entire structure or a component part of the entire structure;

X = total number of unknowns associated with the entire structure or a component part of the entire structure;

then;

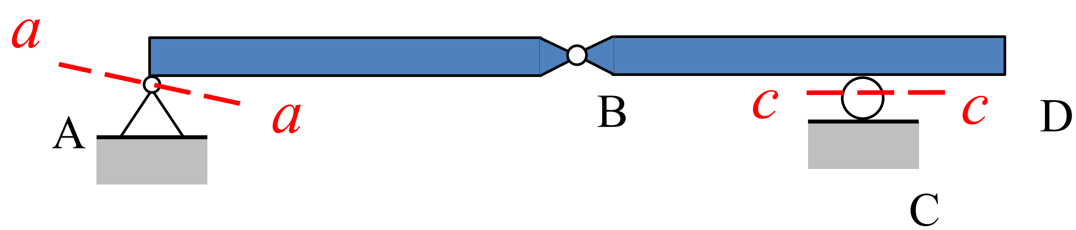
$3n$ = total number of independent equations of equilibrium available to solve for unknowns since there are 3 equations of equilibrium available per FBD of each "piece"

For planar structures, if:

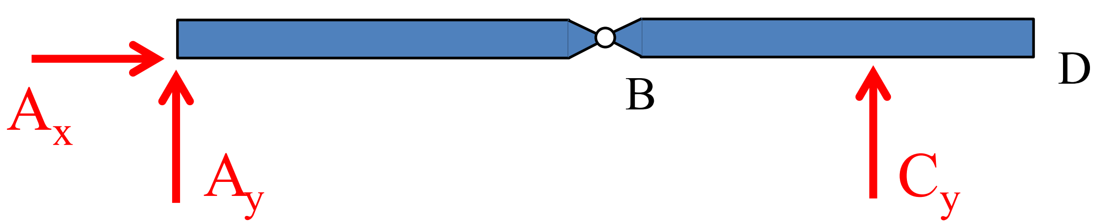
$$X < 3n$$

for the structure or any component part of the structure, then the structure is **Unstable** due to **Partial Constraints**

Example of Instability due to Partial Constraints



FBD of the structure isolated from its supports

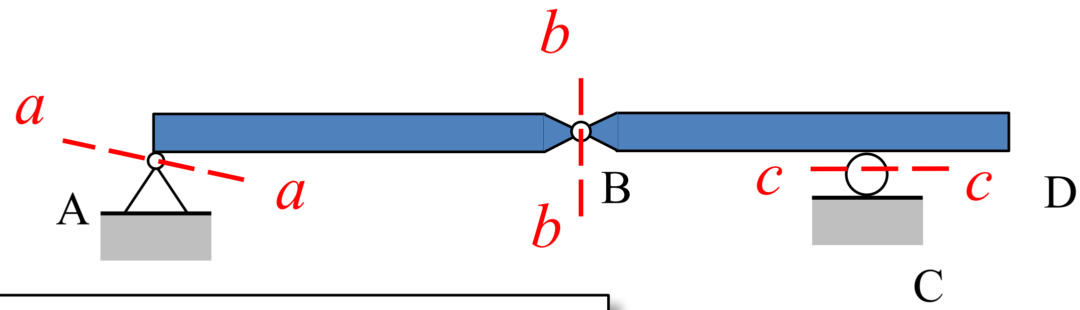


$X = 3$

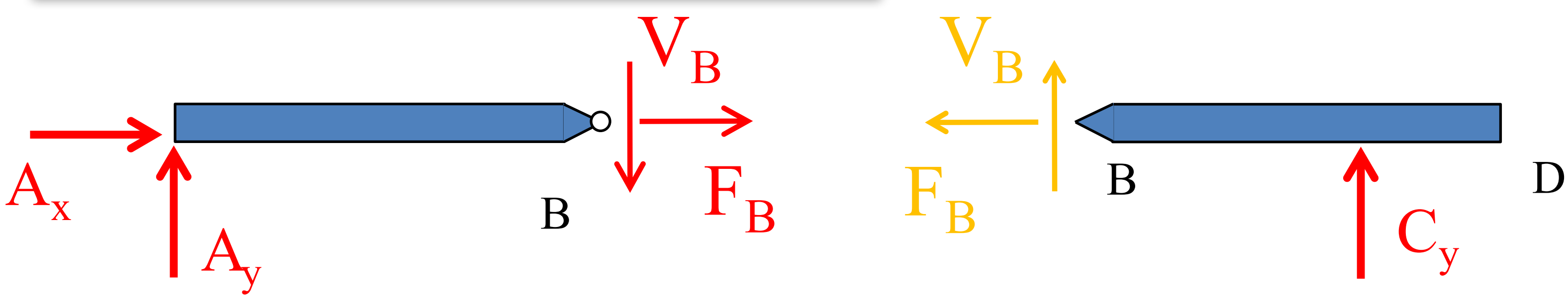
$3n = 3(1) = 3$

OK

Example of Instability due to Partial Constraints



FBDs of component pieces



$$X = 4 + 1 = 5$$

$$3n = 3(2) = 6$$

$$5 < 6$$

Unstable
Partial
Constraints

Assessing Instability Due to Improper Constraints

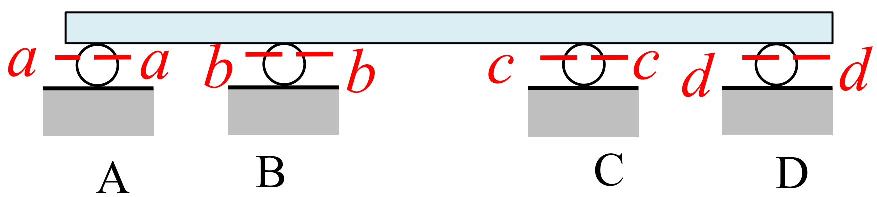
For planar structures, if:

$$X \geq 3n$$

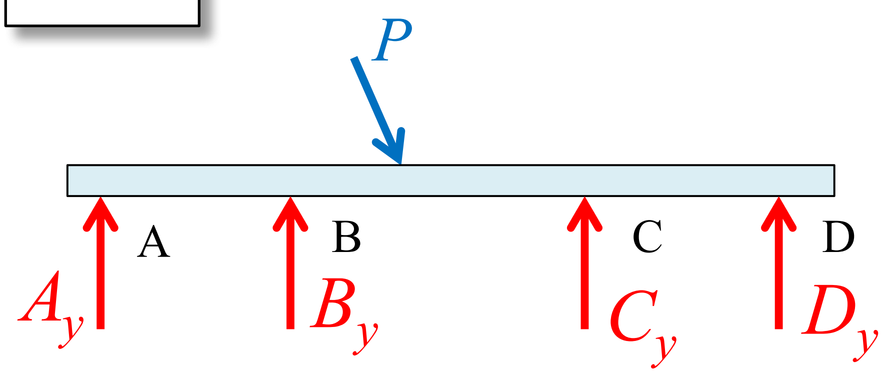
then the structure may be **Unstable** due to **Improper Constraints** if:

A. All of the reactive forces are parallel for the entire structure or any component part of the entire structure;
B. All of the reactive forces are collinear (intersect at one point) for the entire structure or any component part of the entire structure.

Example of Instability due to Improper Constraints



FBD



$X = 4$

$3n = 3(1) = 3$

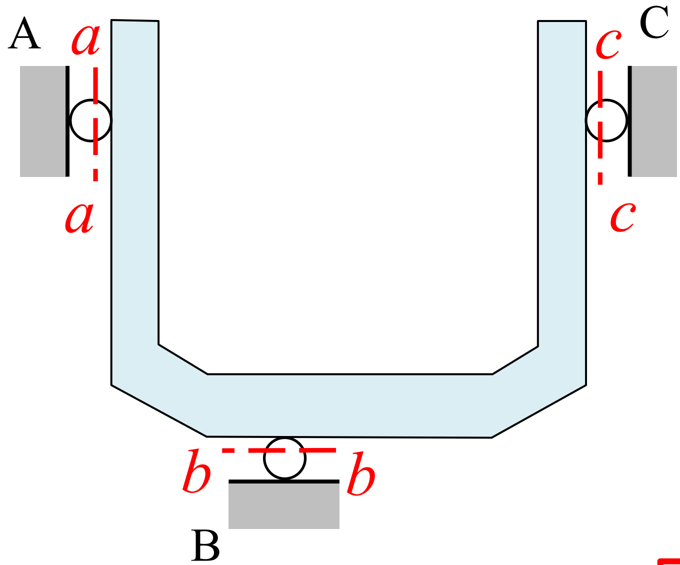
All reactive forces are parallel

$$\overset{+}{\rightarrow} \sum F_x \neq 0$$

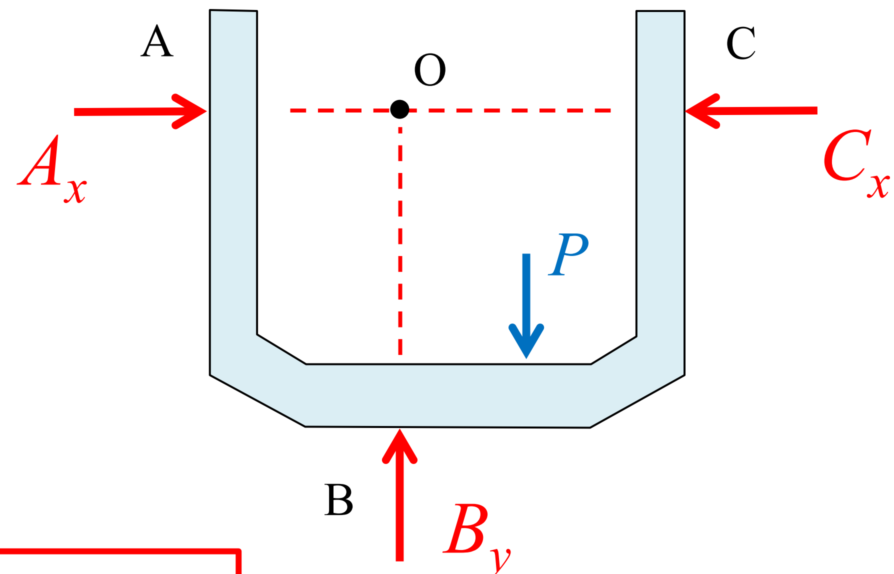
Unstable

Improper Constraints

Example of Instability due to Improper Constraints



FBD



$X = 3$

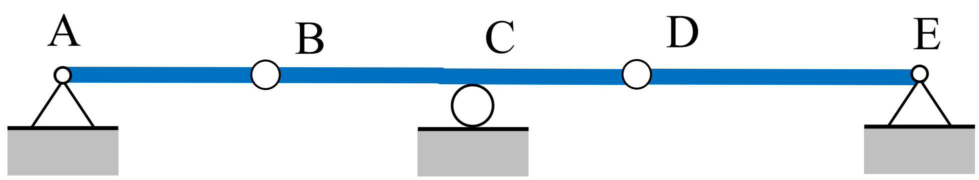
$3n = 3(1) = 3$

$+\circlearrowleft \sum M_O \neq 0$

All reactive forces are concurrent

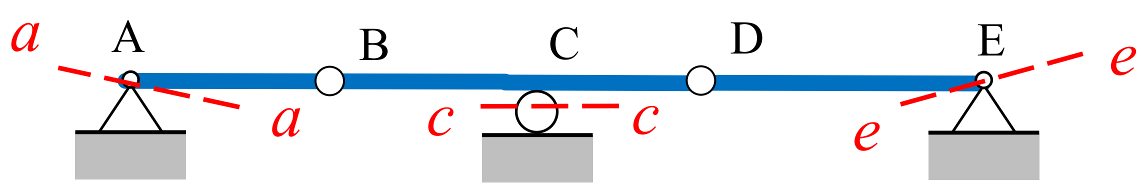
Unstable
Improper Constraints

Example Problem

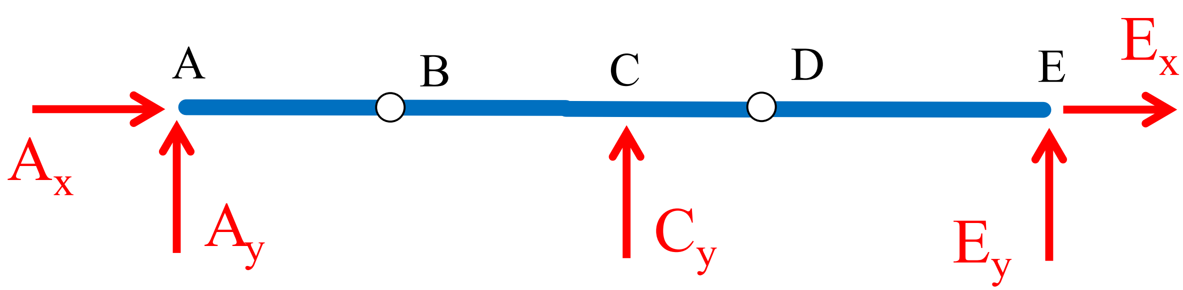


The beam shown has pin supports at A and E, and a roller support at C. There are two internal hinges at B and D as shown. Determine if the beam is stable under all possible loading conditions.

Assess Entire Beam for Stability



FBD of beam cut only at supports

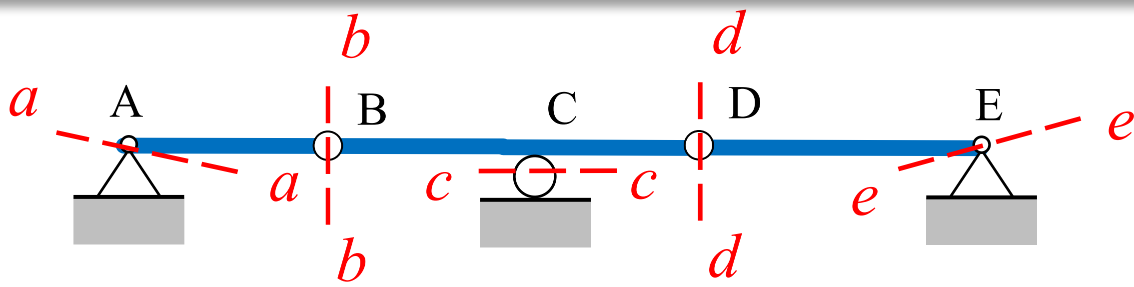


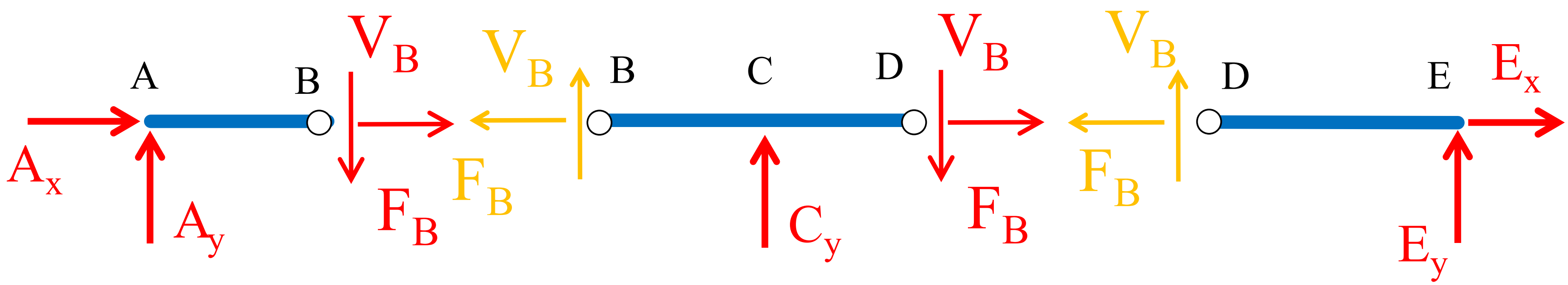
$X = 5 > 3$

Partial Constraints OK for entire beam

Also: **Improper Constraints OK for entire beam**

Examine Component Pieces of the Beam for Stability





Piece 1

$X = 4 > 3$

Piece 2

$X = 5 > 3$

Piece 3

$X = 4 > 3$

Entire Beam

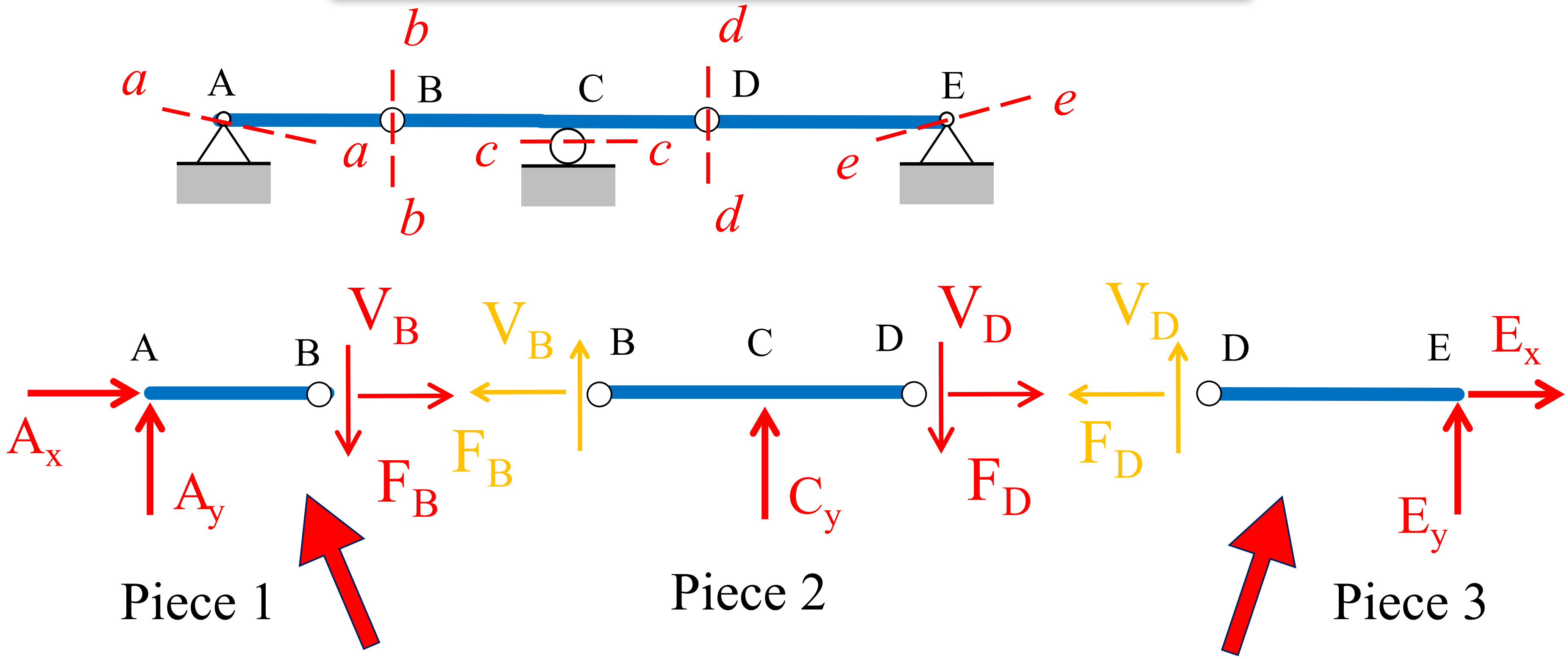
$X = 4 + 3 + 3 = 9$

$3n = 3(9) = 9$

Partial Constraints OK
For entire beam and component pieces

Now check component pieces for improper constraints

Check Component Pieces for Possible Conditions of Improper Constraints

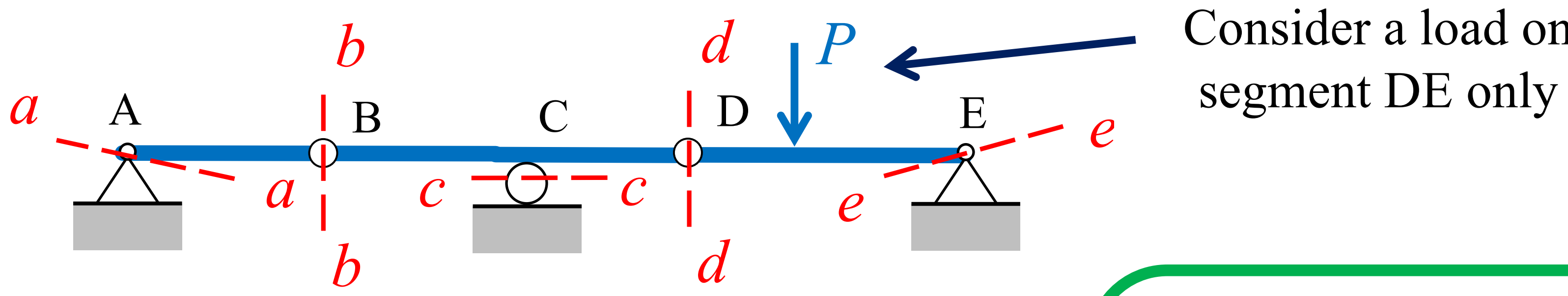


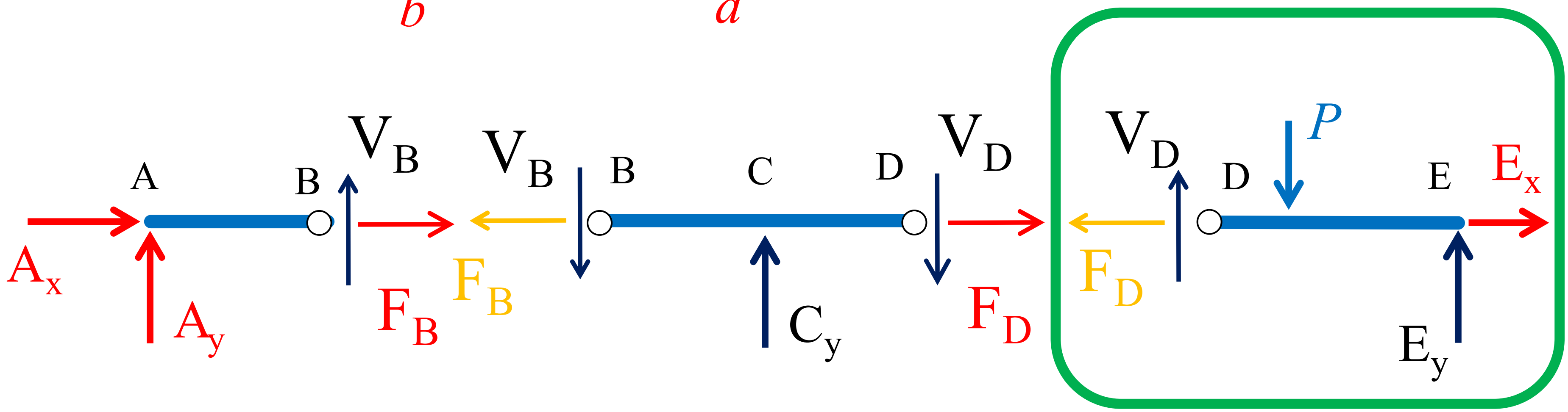
Note if $V_B \neq 0$ with no other load on AB, then $\sum M_A \neq 0$

Note if $V_D \neq 0$ with no other load on DE, then $\sum M_E \neq 0$

Need to check if it is possible to load the beam to cause instability due to improper constraints

Check Loading for Improper Constraints





$\circlearrowleft + \sum M_A \neq 0$

$\circlearrowleft + \sum M_C = 0 \rightarrow V_B$

$\circlearrowleft + \sum M_E = 0 \rightarrow V_D$

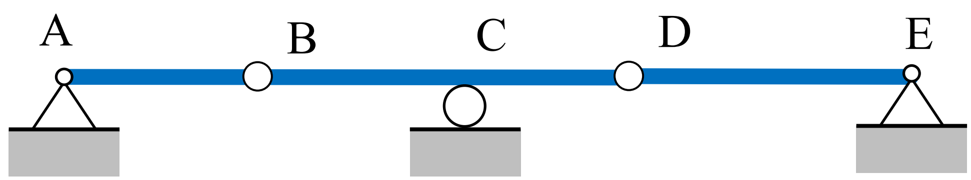
Unstable
Improper Constraints

$+\uparrow \sum F_y = 0 \rightarrow C_y$

$+\uparrow \sum F_y = 0 \rightarrow E_y$

Beam is Unstable

Final Result for Example Problem



Beam is Unstable

Improper Constraints