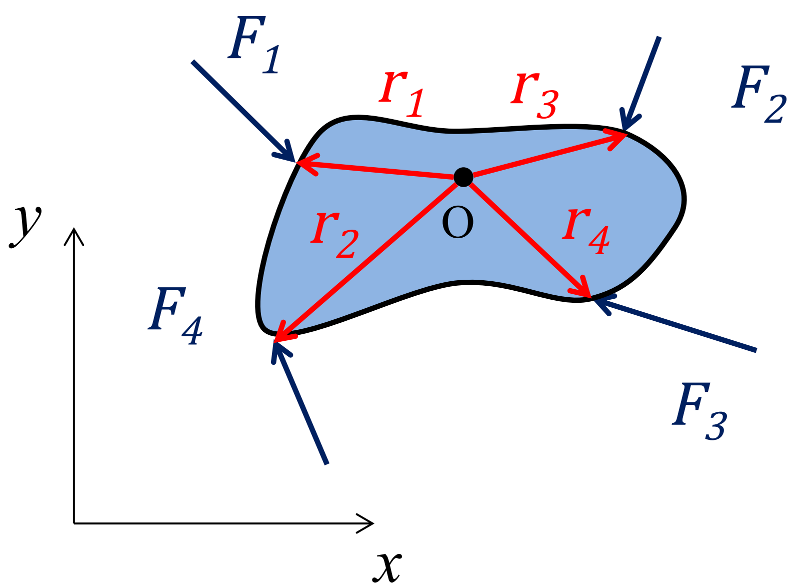
Determinacy for Planar Structures

Steven Vukazich
San Jose State University

Recall for a two-dimensional (planar) body, we can write three independent equations of equilibrium per free-body diagram



Three independent scalar equations of equilibrium

Can take moments about any point

$$\sum F_x = 0 \quad \sum F_y = 0$$

$$\sum M_O = 0$$

Definition of Determinacy of a Planar Structure

A **statically determinate** structure is one that is stable and all unknown reactive forces can be determined from the equations of equilibrium alone.

A **statically indeterminate** structure is one that is stable but contains more unknown forces than available equations of equilibrium.

We will see later in the course that statically indeterminate structures can be solved but require information on the deformation of the structure.

Assessing the Determinacy of a Planar Structure

In order to accurately count the total number of unknowns in a structure we must, at the minimum, cut the structure at the following locations and draw a FBD of each "piece" of the structure:

1. The structure must be cut at all supports (e.g. pins, rollers, fixed supports);
2. The structure must be cut at all locations of known internal force (e.g. internal hinges);
3. The structure must be cut in such a way to "open" all closed rigid loops (a closed rigid loop will be defined later).

Once the structure has been cut at the locations above, the number of "pieces" of the structure will be defined. The total number of unknowns can then be determined once a FBD of each "piece" is drawn.

Assessing the Determinacy of a Planar Structure

Let:

n = total number of "pieces" associated with the entire structure;

X = total number of unknowns associated with the entire structure;

then;

$3n$ = total number of independent equations of equilibrium available to solve for unknowns since there are 3 equations of equilibrium available for the FBD of each "piece"

For stable, planar structures, if:

$$X = 3n$$

then the structure is **Statically Determinate**

Assessing the Determinacy of a Planar Structure

If:

$$X \geq 3n$$

then the structure is **Statically Indeterminate** and the **degree of indeterminacy** is:

$$X - 3n$$

For example, if $X = 8$ and $n = 2$, and the structure is stable, then the structure is **statically indeterminate to the 2nd degree**.

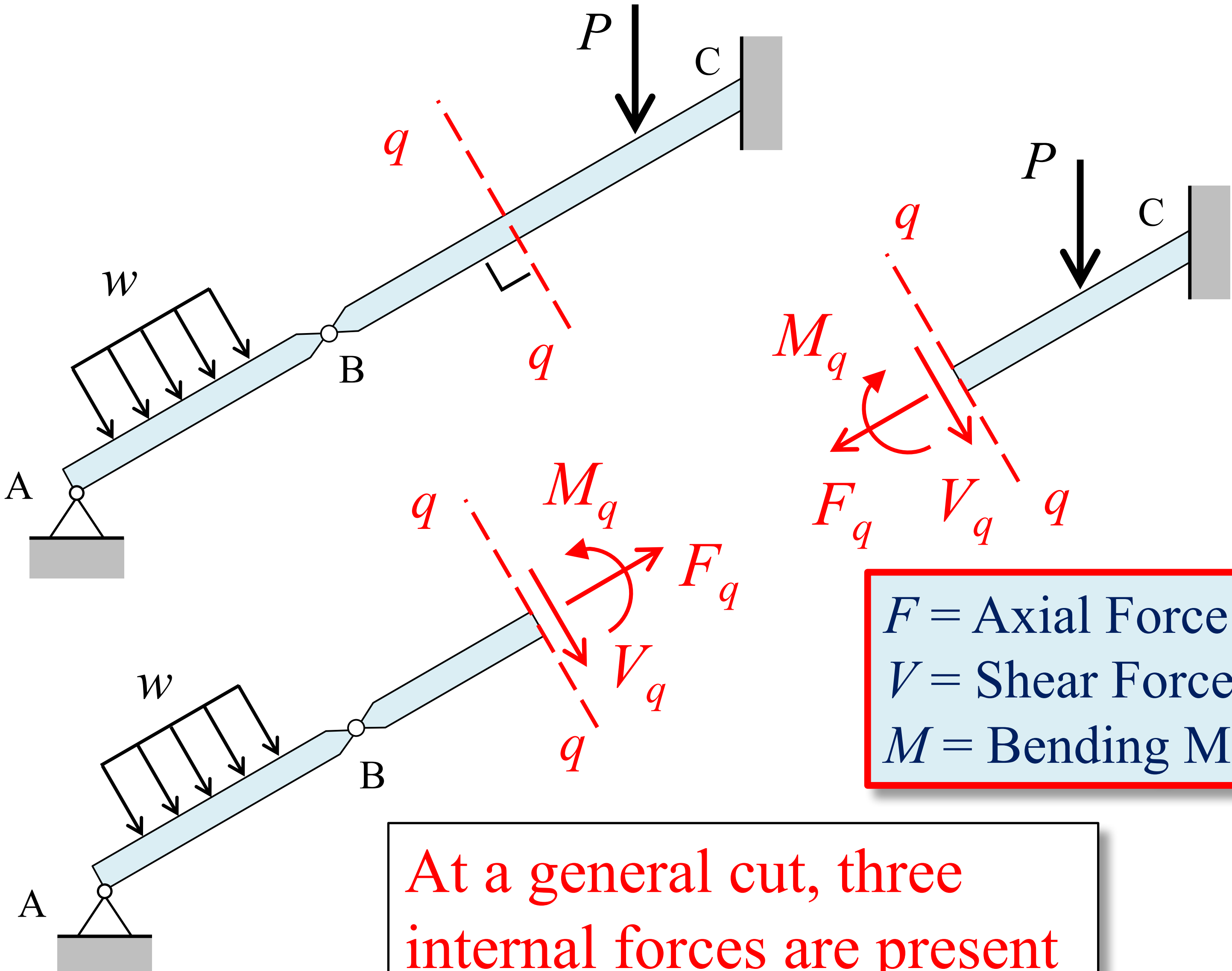
$$8 > 3(2)$$

$$8 > 6$$

and $8 - 6 = 2$

Note that determinacy (and indeterminacy) is only defined for stable structures. We will study how to evaluate structures for stability later.

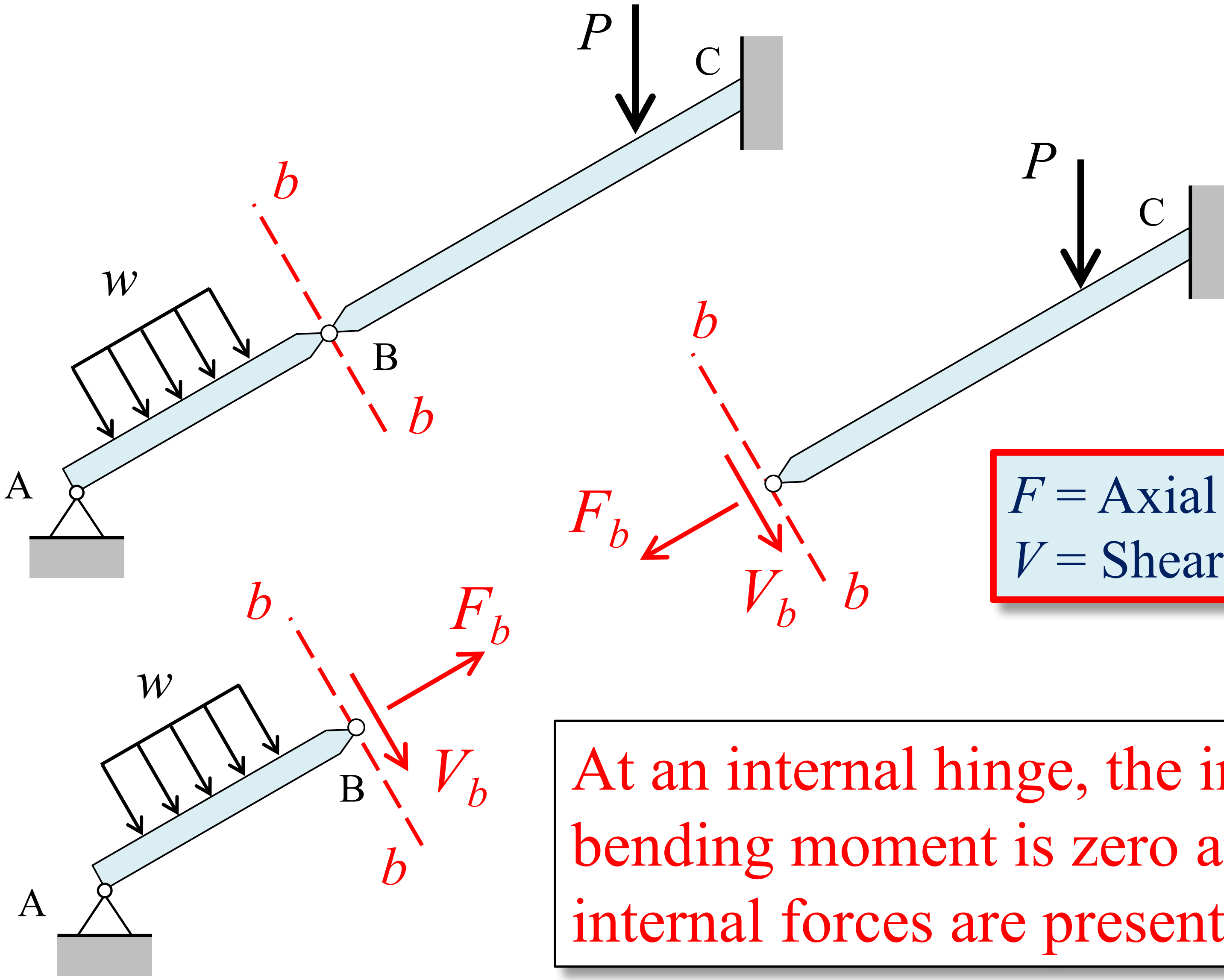
Internal Forces developed in a Planar Structure



F = Axial Force
V = Shear Force
M = Bending Moment

At a general cut, three internal forces are present

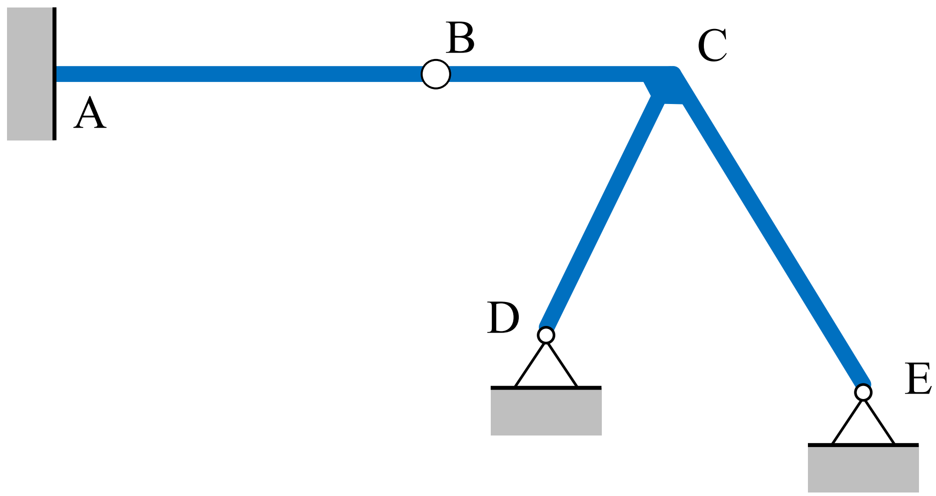
Internal Forces at a Hinge



F = Axial Force
V = Shear Force

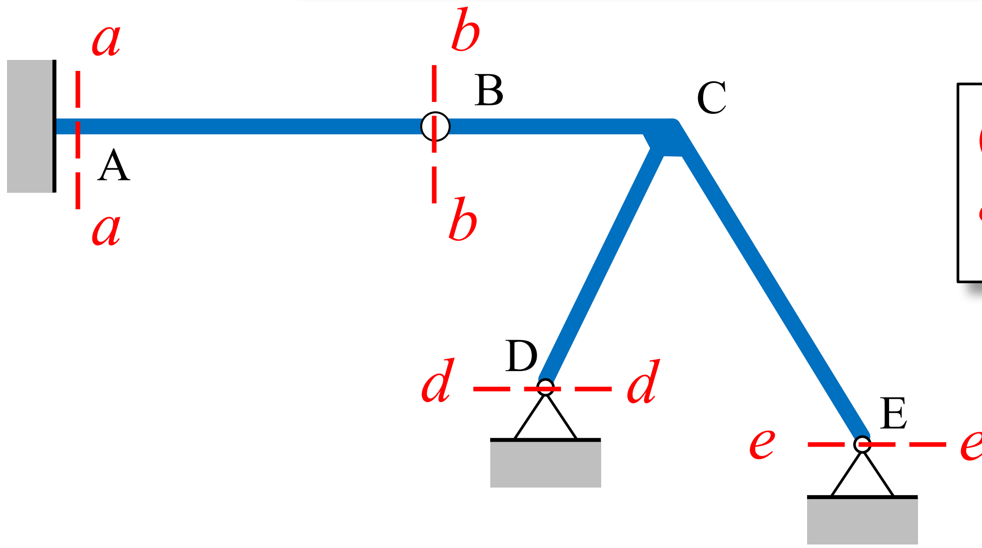
At an internal hinge, the internal bending moment is zero and two internal forces are present

Example Problem



The stable frame shown has a fixed support at A and pin supports at D and E. The frame members are rigidly connected at point C and there is an internal hinge at B. If the frame is subjected to general loading, find the determinacy of the frame.

Example Problem

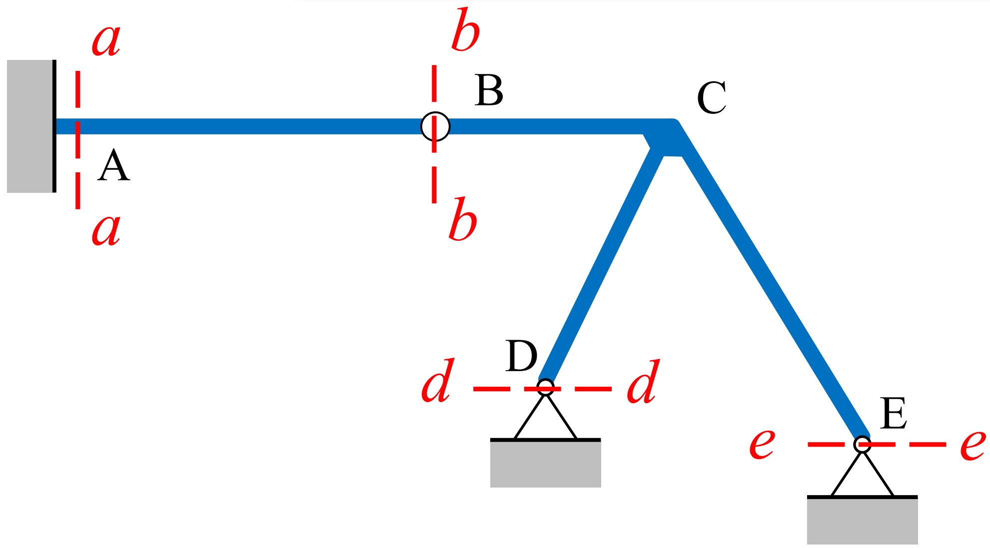


Cuts define two "pieces"

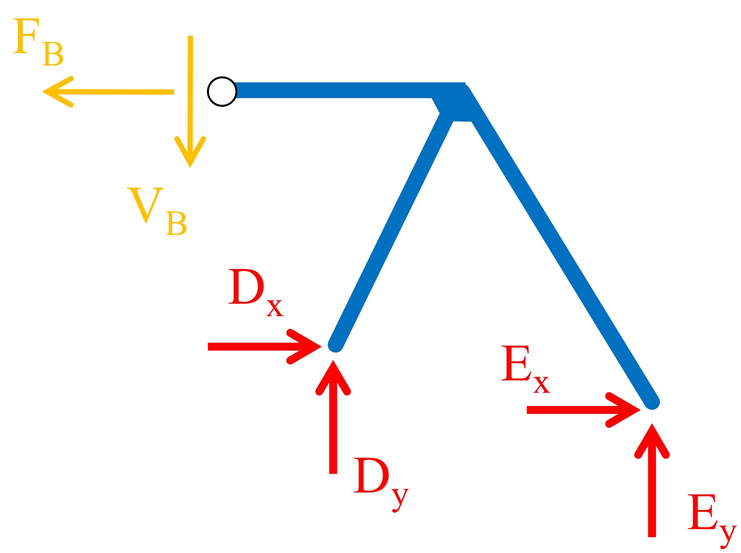
We must, at the minimum, cut the structure at the following locations and draw a FBD of each "piece" of the structure:

1. The structure must be cut at all supports (e.g. pins, rollers, fixed supports); Cuts at A, D, and E;
2. The structure must be cut at all locations of known internal force (e.g. internal hinges); Cut at B;
3. The structure must be cut in such a way to "open" all closed rigid loops; There are not rigid loops in this frame.

Draw FBD of Each Piece



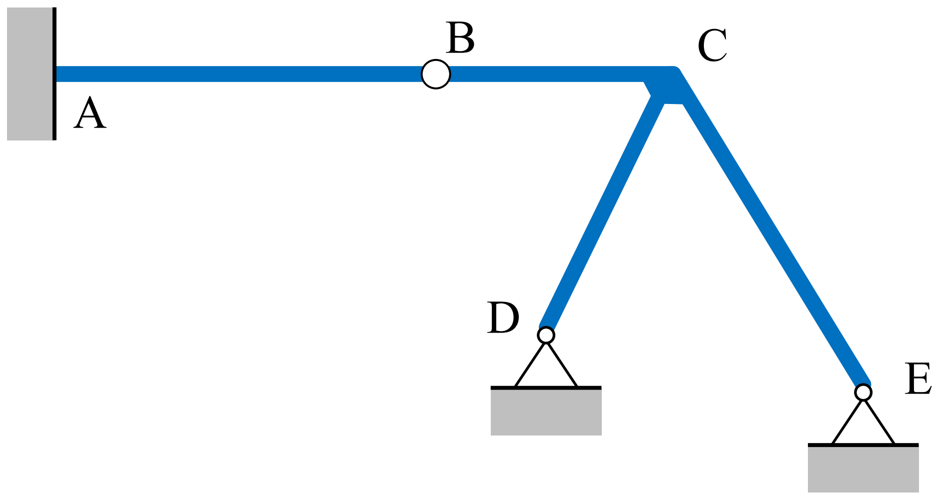
Count total number of unknowns and equations



$$X = 5 + 4 = 9$$

$$3n = 3(2) = 6$$

Final Result



$X = 9$

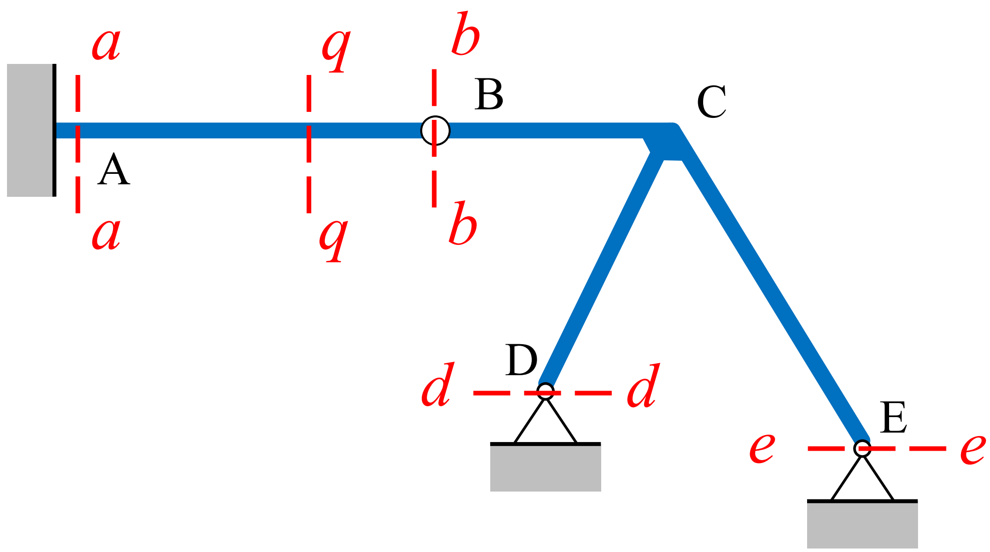
$3n = 6$

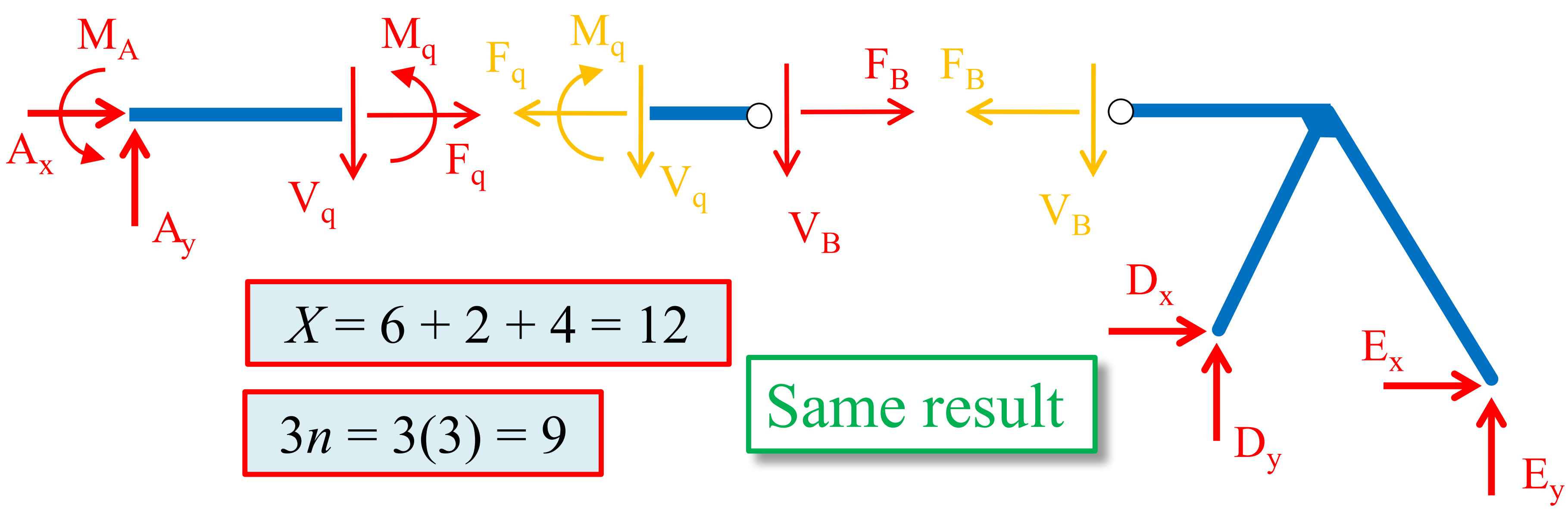
$9 > 6$

$9 - 6 = 3$

Frame is statically indeterminate to the 3rd degree

Note that Cutting the Frame Further Yields the Same Result





$$X = 6 + 2 + 4 = 12$$

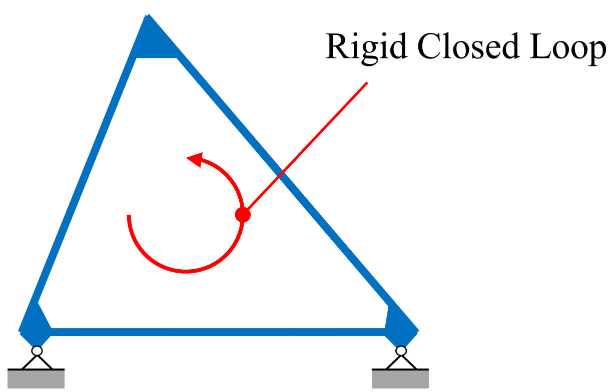
$$3n = 3(3) = 9$$

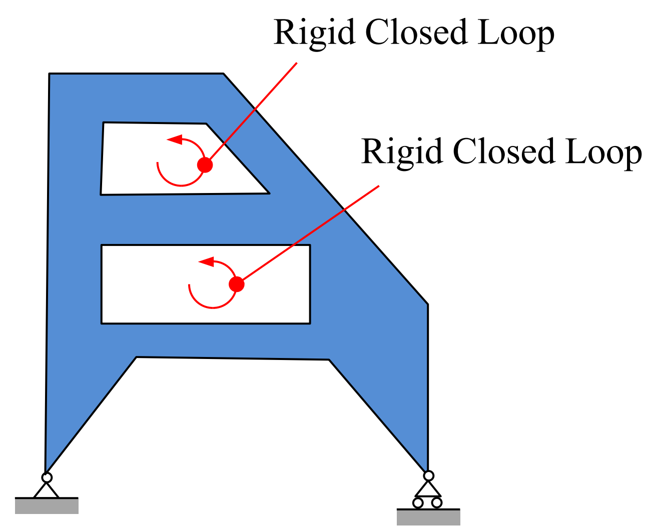
Same result

Rigid Closed Loops

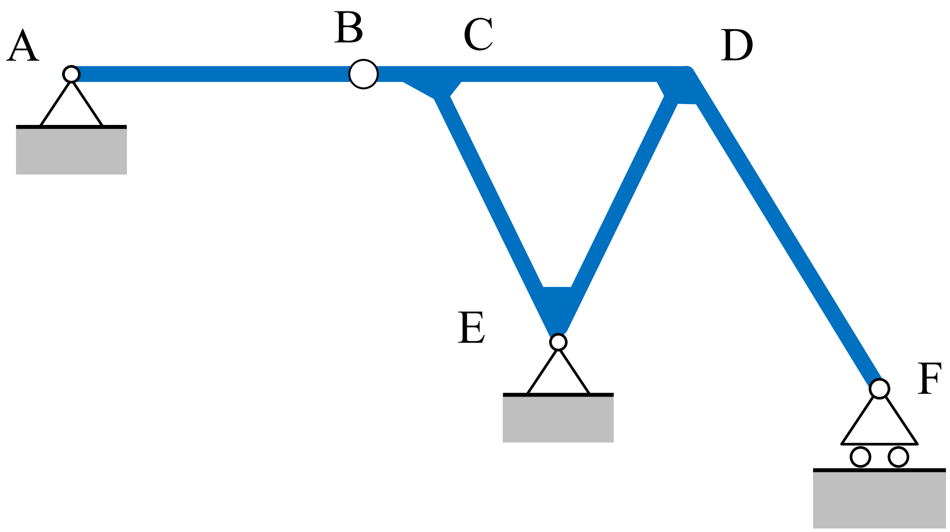
If we can circle a loop in a planar structure and all of the connections around the loop are rigid, then that defines a rigid closed loop and we must cut through the loop in order to accurately access the determinacy of the structure.

Examples of structures that contain rigid closed loops.



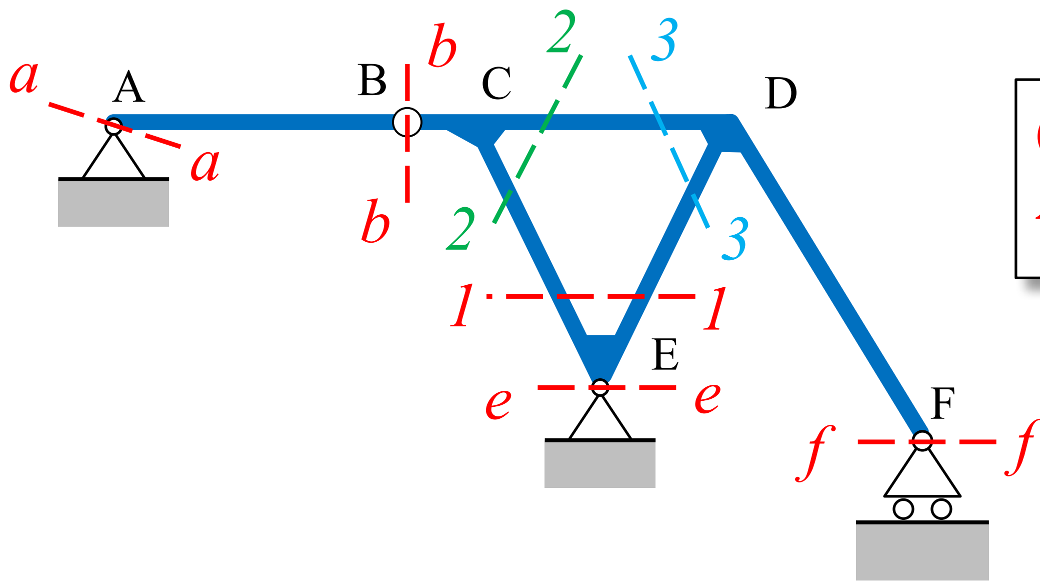


Another Example Problem



The stable frame shown has a roller support at F and pin supports at A and E. The frame members are rigidly connected at point C, D, and E and there is an internal hinge at B. If the frame is subjected to general loading, find the determinacy of the frame.

Another Example Problem

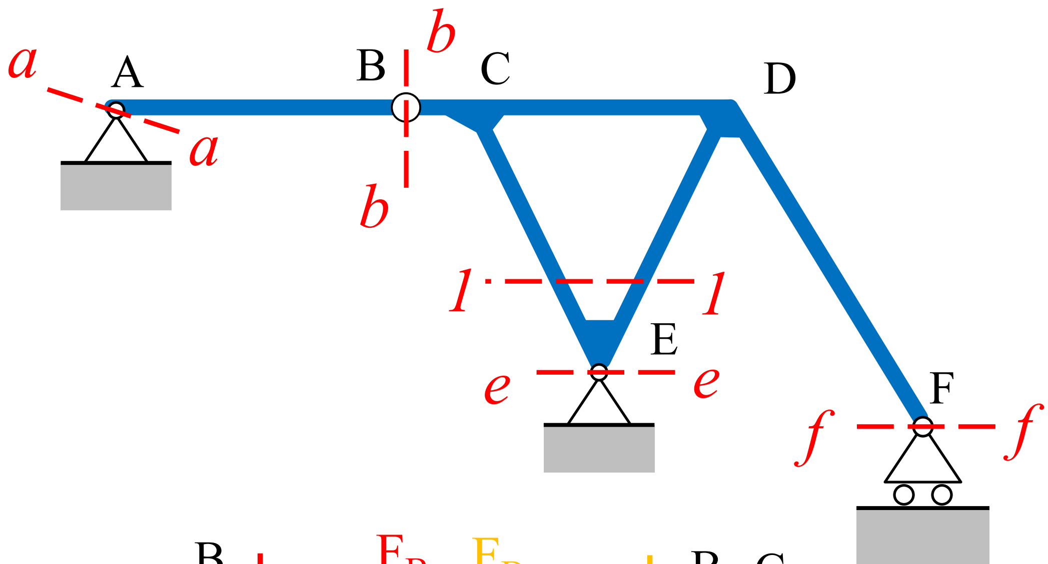


Cuts define three “pieces”

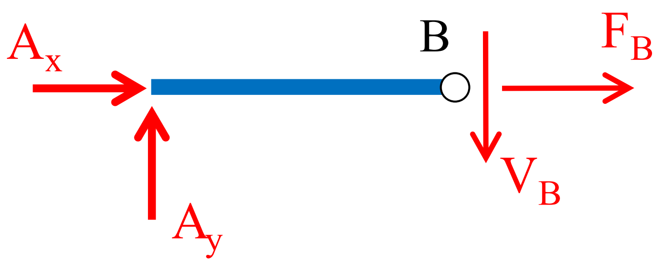
We must, at the minimum, cut the structure at the following locations and draw a FBD of each “piece” of the structure:

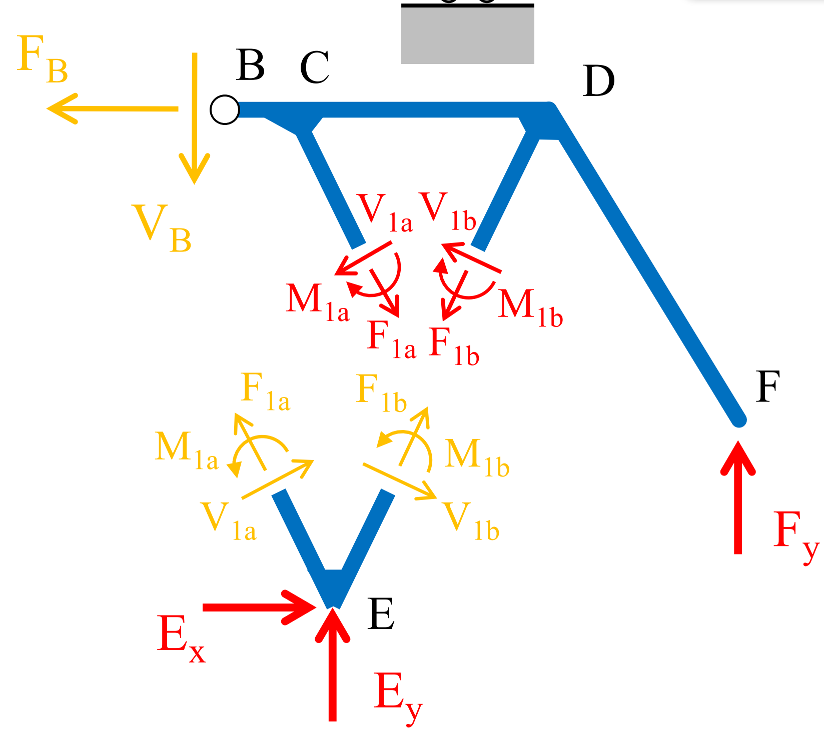
1. The structure must be cut at all supports (e.g. pins, rollers, fixed supports); Cuts at A, E, and F;
2. The structure must be cut at all locations of known internal force (e.g. internal hinges); Cut at B;
3. The structure must be cut in such a way to “open” all closed rigid

Draw FBD of Each Piece



Count total number of unknowns and equations

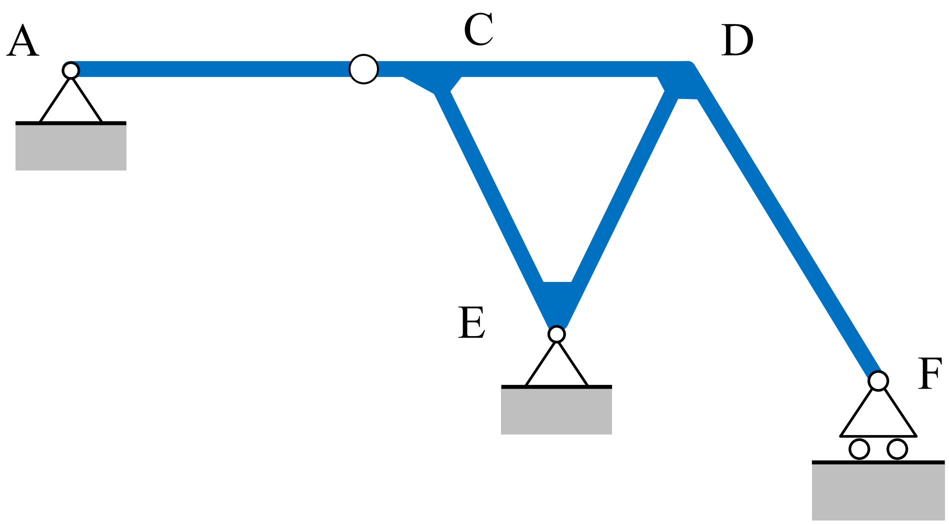




$$X = 4 + 7 + 2 = 13$$

$$3n = 3(3) = 9$$

Final Result



$X = 13$

$3n = 9$

$13 > 9$

$13 - 9 = 4$

Frame is statically indeterminate to the 4th degree

Determinacy is Only Applicable to Stable Structures

As noted initially, determinacy is defined only for stable structures. The process for assessment is as follows:

Check stability → Check determinacy → $X = 3n$ **Determinate**

Check stability → Check determinacy → $X > 3n$ **Indeterminate**

Check stability → **Unstable**

In the next unit, we will study how to assess structural stability