

Defining, teaching, and assessing problem solving skills

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ABSTRACT:

Problem solving skills have always been important in many professions. However, ABET EC 2000 [1] recently put a new focus on these skills in engineering education with outcome 3e, which states that engineering graduates must “*have an ability to identify, formulate and solve engineering problems*”. Problem solving is defined as a process, used to obtain a best answer to an unknown, or a decision subject to some constraints. Problem solving is not the same as textbook exercise solving, which is very common in engineering curricula. The paper first defines engineering problem solving and in particular what it means to “*identify and formulate*” a problem. This definition will set the stage for identifying the skills students need to acquire and the attributes they must possess to be classified as competent problem solvers. Next, the paper introduces course design elements that help students master these skills. Finally, the paper presents and analyzes data on student performance in these types of problems.

INTRODUCTION

Engineers by definition are problem solvers. Whether they are involved in analytical, experimental, computational or design work, engineers solve problems. Yet, real world problems tend to be quite different than most exercises found in engineering texts. While these exercises make an important first step in helping students bridge the gap between theory and application, they do not provide the complexity and depth necessary to master problem solving skills. Many studies have found that engineering graduates, even though they solve more than 2,500 exercises in their undergraduate work, lack the essential problem solving skills needed to tackle real world problems [2].

In this paper we differentiate between problem solving and exercise solving, which is very common in engineering curricula. Table 1 shows the main differences between the two. Items 7 and 2 suggest that in real world problems engineers must first define the problem itself. They must decide what exactly they need to calculate, to answer the question. This may involve translating a need expressed in layman’s jargon into engineering terms. Moreover, items 4 and 1 suggest that in real world problems engineers have to formulate the problem. They must decide what is the appropriate theory applicable to the given situation and what approach they will follow to calculate the unknown quantities. This step requires additional assumptions (modelling), which allow complicated fundamental equations to be reduced into simplified forms that can be solved.

Students who train mostly in exercise solving tend to develop a serious handicap. They rely heavily on solutions they have seen before, rather than working directly from first principles. Thus, a problem with brand new context presents a formidable challenge to them.

DEFINITION OF PROBLEM SOLVING SKILLS

Woods et al [2] assert that students who are problem solvers exhibit the following attributes:

1. Are willing to spend time reading, gathering information and defining the problem.
2. Use a process, as well as a variety of tactics and heuristics to tackle problems.
3. Monitor their problem-solving process and reflect upon its effectiveness.
4. Emphasize accuracy rather than speed.
5. Write down ideas and create charts / figures, while solving a problem.
6. Are organized and systematic.
7. Are flexible (keep options open, can view a situation from different perspectives / points of view).
8. Draw on the pertinent subject knowledge and objectively and critically assess the quality, accuracy, and pertinence of that knowledge / data.
9. Are willing to risk and cope with ambiguity, welcoming change and managing stress.
10. Use an overall approach that emphasizes fundamentals rather than trying to combine various memorized sample solutions.

It is interesting to note that these attributes come from both the cognitive (2, 3, 5, 8, 10) and the affective (1, 4, 6, 7, 9) domain [3,4]. This observation suggests that students need to develop first certain attitudes before they acquire the skills necessary to tackle open-ended problems. Moreover, it is obvious from the description of these attributes that level 4 is the minimum level of competence required in both the affective [3] and the cognitive [4] domain to perform as an expert problem solver.

Table 1. Differences between problem solving and exercise solving.

	Problem Solving	Exercise Solving
1.	Involves a process used to obtain a best answer to an unknown, subject to some constraints.	Involves a process to obtain the one and only right answer for the data given.
2.	The situation is ill defined. There is no problem statement and there is some ambiguity in the information given. Students must define the problem themselves. Assumptions must be made regarding what is known and what needs to be found.	The situation is well defined. There is an explicit problem statement with all the necessary information (known and unknown).
3.	The context of the problem is brand new (i.e., the student has never encountered this situation before).	The student has encountered similar exercises in books, in class or in homework.
4.	There is no explicit statement in the problem that tells the student what knowledge / technique / skill to use in order to solve the problem.	Exercises often prescribe assumptions to be made, principles to be used and sometimes they even give hints.
5.	There may be more than one valid approach.	There is usually one approach that gives the right answer.
6.	The algorithm for solving the problem is unclear.	A usual method is to recall familiar solutions from previously solved exercises.
7.	Integration of knowledge from a variety of subjects may be necessary to address all aspects of the problem.	Exercises involve one subject and in many cases only one topic from this subject.
8.	Requires strong oral / written communication skills to convey the essence of the problem and present the results.	Communication skills are not essential, as most of the solution involves math and sketches.

COURSE DESIGN TO ADDRESS PROBLEM SOLVING SKILLS

To first step in teaching problem solving skills is to adopt a methodology for approaching open-ended problems. There are many such approaches available in the literature [5,6]. We selected Wood's method [6] because it was developed specifically for engineers. The steps of this methodology are as follows:

0. Engage / Motivation

- I can do it!
- I want to do it!

1. Define the problem

- Define what the problem states
- Sketch the problem (if appropriate)
- Determine the given information
- Determine constraints
- Define criterion for judging final product

2. Explore the problem

- Determine the real objective of the problem
- Examine issues involved
- Make reasonable assumptions
- Guestimate the answer

3. Plan the solution

- Develop a plan to solve the problem
- Map out sub-problems
- Select appropriate theory, principles, approach
- Determine info that needs to be found

4. Implement the plan

5. Check the solution (units, accuracy?)

6. Evaluate / Reflect

- Is it reasonable? Does it make sense?
- Were the assumptions appropriate?
- How does it compare to guestimate?
- If appropriate, ask the question: is it socially / ethically acceptable?

The following examples of open-ended problems from fluid mechanics, thermodynamics, and heat transfer show how this method can be applied.

A. Fluid Mechanics

The party is over and it is raining hard. Your car is parked a couple of blocks away. The way to your car is open, exposed to the rain. You are wearing your new, designer clothes. You just got the first monthly statement and it hurts. You want to make sure you soak them as little as possible. You have no umbrella. You are getting ready to run as hard as you can when all of a sudden, you start doubting whether this is the best way to save your clothes. Should you walk instead? The decision is too important to leave to chance. Besides, you are an engineer. You walk back into the building, pull out a pencil and a piece of paper and start looking for the right answer...

1. Define the problem: Students realize that the criterion for deciding whether to walk or run will be the amount of water absorbed by their clothes in the two cases: (a) walking and (b) running the distance from the building to their car. There are absolutely no numbers given in this problem. Students need to translate the distance (2 blocks) into meters (approximately 200 m) and the "heavy rain" into a number of droplets per unit volume (m^3). The constraints are that they have no umbrella, and the way to their car is open, exposed to the rain.

2. Explore the problem: Students realize they need to calculate either the volume (m^3) or the mass (kg) of water absorbed by their clothes while walking / running to their car. They realize they will get wet mostly on the top and the front of their bodies. One of the main issues in this problem, which makes it different from other problems they have seen in fluid mechanics, is that the water flow due to the rain is not continuous. Hence, they need to estimate a flow rate Q (m^3/s) from the number of droplets per unit volume, the vertical speed and the volume of each droplet. All three of these parameters need to be assumed. Additional assumptions involve the walking and running speed of the person. Moreover, to simplify the problem they assume

that there is no wind, so the rain falls vertically at a constant speed. At this point students guess that it is probably better to run than to walk, however, their estimates of how much water is actually absorbed into their clothes in each case are not always realistic.

3. Plan the solution: Students draw a control volume around a human body of typical dimensions. They divide the problem in two parts: calculating (a) how much water enters the control volume from the top, and (b) how much water enters the control volume from the front. They define quantities such as:

ϕ = # of droplets per unit area (m^2), per unit time (sec), and

ψ = # of droplets per unit volume (m^3)

They write an expression for the flow rate (m^3/s) through the top surface of the control volume as:

$$Q_{\text{top}} = A_{\text{top}} \phi v_{\text{droplet}}$$

where v_{droplet} is the assumed volume of a single rain droplet. Similarly, the flow rate through the front surface of the control volume can be written as:

$$Q_{\text{front}} = A_{\text{front}} V_{\text{run/walk}} \psi v_{\text{droplet}}$$

Further examination of these two expressions reveals that while the volume of water absorbed through the top depends on time, the volume of water absorbed through the front depends only on the distance covered and the dimensions (height and width) of the control volume. This is so because there is a fixed number of droplets at any given time in the space traversed by the control volume, which they will be absorbed regardless of how fast the person moves through this space. This point is usually a revelation for most students.

4. Implement the plan: Students simply substitute into their equations the values assumed for each quantity and carry out the calculations.

5. Check the solution: Students check for (a) accuracy of their calculations and (b) correctness of units. This is especially important when new quantities are introduced, such as ϕ and ψ .

6. Evaluate / Reflect: Students check whether their answer makes sense. For example, if they estimate the amount of water absorbed to be 0.5 kg that is reasonable. On the other hand, if their estimate turns out to be 15 kg (too large) or 0.5 g (too small) it would not be acceptable. In some cases students make unrealistic assumptions (ex. $\psi = 10^6$ droplets / m^3) resulting in a huge volume of water absorbed by their clothes. At this point they need to recognize this, go back, revise their assumptions, and rework their solution to get a more reasonable answer.

B. Thermodynamics

A nuclear power plant whose dimensions and initial conditions are given, develops a break in its main steam line. Sensors detect the accident and cause the reactor to shut down. Because the reaction cannot be stopped instantaneously, there is some residual energy transfer from the fuel rods into the reactor vessel that decays to zero after a given amount of time. The high-pressure steam in the reactor vessel leaks out of the steam line and starts to fill the primary containment compartment known as the drywell. The mixture in the drywell then enters the secondary containment compartment known as the wetwell, which contains a large mass of subcooled water. The water is used to condense steam from the accident and thus limit the pressure response. Is this design sufficient for keeping the pressure below design limits in the event of this accident?

1. Define the problem: This problem incorporates many technical topics discussed in an advanced thermodynamics course but leaves the details of the approach and implementation up to the student. There are three subsystems in this problem: the reactor vessel, the drywell, and the wetwell. Students realize that the mass and internal energy (in addition to all other properties) vary with time; therefore, unsteady mass and energy balances will be required to determine the pressure response in each subsystem. The maximum pressure in the drywell and the wetwell can then be compared to the design limit to determine if the criterion is met. The constraints include pressure driven flow from the reactor vessel to both containment compartments and pump flow from the wetwell back to the reactor vessel.

2. Explore the problem: The initial conditions and the criterion for judging the final answer are stated, but the applicable assumptions and available tools are not. The unsteady mass and energy balances require the evaluation of derivatives that are a function of multiple interdependent thermodynamic properties. In addition, determination of properties for a transient problem can be tedious if there are no applicable equations of state. Some numerical tools available include a simultaneous equation solver with property look-ups (EES, F-Chart Software), and Microsoft Excel with the thermophysical property module. It might be a reasonable assumption to assume that thermodynamic equilibrium exists in each compartment at each time step, such that properties can be determined. The reactor vessel can be modelled as a homogeneous mixture of steam and water. The drywell is a non-reacting mixture of air and steam. The wetwell is a two-phase system with an air and water vapor mixture above a subcooled liquid water pool. At thermodynamic equilibrium, the relative humidity is 100%. Because the mass of water vapor in the airspace is insignificant compared to the wetwell pool, computations are simplified and the error introduced is minimal if the humidity is assumed to be 0%. It can be reasoned that at steady state, much of the energy initially in the reactor vessel will end up in the wetwell pool. The students may guess that a maximum pressure will be reached at some time after the start of the accident, and will then subside to the steady state solution.

3. Plan the solution: Students write down the equations for mass and energy conservation for each subsystem. The mass flow rates are then modelled as proportional to the time dependent pressure difference between two subsystems. The derivatives in the mass and energy balances can be evaluated numerically with EES or Excel. Once the mass and internal energy of each subsystem is known at each time step, the corresponding enthalpy, pressure, and temperature can be evaluated.

4. Implement the plan: Students enter their equations into a numerical solver and carry out their calculations. Some level of programming skills and in particular systematic debugging skills are required for this assignment. Graphs showing the time-dependent solutions are generated.

5. Check the solution: The solution can be compared to several benchmarks to ensure it is reasonable. Does the steady state solution make sense, and is it what was expected? Is the total mass in the system constant as dictated by control mass analysis? Does the total energy of the system increase by the amount of residual energy transfer from the fuel rods?

6. Evaluate / Reflect: Students check if the maximum pressure in the drywell and wetwell exceeds the design pressure as requested

by the assignment. If not, they reflect on differences between the model generated for the assignment and additional emergency response systems in actual power plants that may work to further reduce pressure responses.

C. Heat Transfer

Your job is to design an experiment to determine the thermal conductivity (k) of a solid metal rod. The rod has a diameter of 7.0 cm and a length of 12 cm. The material is unknown. You may machine this rod any way you wish for your design. You may assume that the rod has a thermal conductivity in the range of 10-200 W/mK. Make sure that your uncertainty in temperature provides no more than 10% uncertainty in your calculated value of k . For ideas about equipment, you may wish to look at www.omega.com. Deliverables include a memo summarizing your design, a list of the equipment that you will use, a detailed sketch of your design, including dimensions, and a page giving sample calculations.

1. Define the problem: The students must find k . Since k can change with temperature, they must realize that these changes tend to be small for metals over a limited temperature range. Therefore, they are looking for k at some average temperature. The only constraints deal with the uncertainty in temperature and the shape of material given to them.

2. Explore the problem: After some brainstorming, most students realize that this problem can be solved either by assuming that the metal acts like a fin or else that heat transfer through the metal is one-dimensional (other solutions are also possible). Most students use the latter assumption. For homework exercises, students are usually told when they can assume that heat transfer is one-dimensional. Here they must design their experiment such that this assumption is valid. Students must also realize that the greater the temperature difference from one side of the metal to the other, the less the temperature uncertainty will affect the final value of k .

3. Plan the solution: For the 1-D (one dimensional) heat transfer solution, Fourier's Law applies: $\dot{Q}_x = -kA dT/dx = -kA\Delta T/\Delta x$. Here \dot{Q} is the applied rate of heat transfer (Watts in the SI system), A is the cross-sectional area of the block, ΔT is the temperature difference between two thermocouples located towards either side of the block, and Δx is the distance between the thermocouples.

4. Implement the solution: Students must provide enough insulation that heat loss through the sides of the block is negligible compared to heat loss from the end for the 1-D assumption to hold. A fan or cold water supply should be added to enhance heat loss from the end. They must include a heat source, such as a resistance heater, that provides a value of \dot{Q} that can be measured accurately and applied to the cylinder base with negligible losses. \dot{Q} must be large enough that ΔT is large enough for the uncertainty in temperature to have only a small effect on the final value of k . They must make sure that their thermocouples accurately measure the cylinder temperature; many of the groups use a thermally conductive paste or epoxy to help achieve this goal.

5. Check the solution: Sample calculation will prove whether or not the uncertainty in temperature has a small effect. Quick

calculations comparing heat loss through the insulation to heat loss from the end can justify the 1-D assumption.

6. Evaluate/Reflect: Although students do not need to build their apparatus, they need to determine whether or not their design is practical to implement. Can their apparatus be cheaply built? Is the rate of heat transfer small enough that it can be applied with readily available resistance heaters or power supplies? Are the temperatures small enough not to burn up their insulation? If not, they must revisit their design.

ASSESSMENT OF STUDENTS' PROBLEM SOLVING SKILLS

Open-ended problems are usually assigned in teams because research has shown that cooperative learning stimulates higher order thinking [7], which is a must for open-ended problems. Table 2 summarizes the performance of the students in the rain problem (fall 2001 and 2003) and the thermal conductivity problem (fall 2003).

Table 2. Student problem solving performance in fluid mechanics and heat transfer.

	Fluid Mechanics Fall 2001	Fluid Mechanics Fall 2003	Heat Transfer Fall 2003
	<i>Rain problem</i>	<i>Rain problem</i>	<i>Thermal conductivity problem</i>
Score	N=28	N=46	N=50
70% or higher	8 (29%)	29 (63%)	31 (62%)
60 – 69%	4 (14%)	0 (0%)	10 (20%)
50 – 59%	6 (21%)	3 (7%)	1 (2%)
lower than 50%	10 (36%)	14 (30%)	8 (16%)

In 2001 the rain problem was briefly introduced in class. Students worked in teams, without any interaction with the instructor, except during the presentation of their solution in class. In 2003 students received more guidance through an in-class discussion. However, the methodology presented here was not given to them, until after they presented their solutions in class. All the teams who received high scores spent a considerable amount of time interacting with the instructor asking questions, checking their models, assumptions, and results before turning in their final report. On the other hand, it became obvious that students who received low scores had not spent enough time on the problem.

The steps that presented the most difficulty for students were the following:

- Making a reasonable assumption for ψ (# of droplets per unit volume).
- Translating the non-continuous rain flow into water flow rate (Q) through the top and the front of the assumed control volume.
- Making a reasonable guesstimate of how much water is absorbed into their clothes.
- Checking for correctness of units in the parameters they calculated.
- Recognizing that their answer (kg of water absorbed into their clothes) was not reasonable. This is a direct consequence of the fact that they could not guesstimate the answer.

- Communicating the essence of the problem, the approach they had chosen, and the significance of their results.

The additional guidance they received in class in relationship to these steps, explains why their scores were significantly better in fall 2003. It is expected that an example problem presented in class, illustrating how to apply the six steps of the problem solving methodology, will further improve their performance in future course offerings.

The thermal conductivity problem was given for the first time in fall 2003. Students had difficulties in two main areas: justifying their assumptions and ensuring less than 10% uncertainty in k due to the temperature measurement uncertainty. Most students assumed that heat loss through their insulation was negligible but provided no calculations to prove that this was true. And despite a discussion in class of how to minimize experimental uncertainty, few students addressed the issue at all. Only one group addressed both of these issues correctly. Students who received a score of less than 50% had major problems with their designs, such as assuming 1-D heat transfer when it clearly was not 1-D or using steady-state equations for a transient experimental set-up. Some students also did not adequately reflect on their solution and thus ended up with designs that worked on paper but would never work in reality (ex. the design that one group came up with, would have required a temperature difference of 20,000°C between two thermocouples if the metal thermal conductivity had been at the low end of the specified range.)

Table 3. Degree of completion for the nuclear reactor problem.

	Stage 1: Outline approach, list assumptions, develop a model	Stage 2: Implement a solution & evaluate the results
Completed successfully	0 (0%)	8 (47%)
Attempted calculation past the initial state	1 (5%)	1 (5%)
Calculated initial state only	16 (95%)	7 (48%)
Remarks	Significant implementation and conceptual errors evident in most submissions	Minor implementation errors seen in 23% of complete solutions

The nuclear reactor problem was assigned for the first time in fall 2003 and was graded in two stages. In the first stage, students were given two weeks to produce a set of equations describing their thermodynamic model, justify their assumptions, and implement their solution. However, grading in the first stage was weighted more heavily towards modelling. The first submission was followed by an in-class discussion on the pros and cons of various assumptions but did not specify which approach should be taken. Subsequently, students were given two more weeks to make further progress on the assignment. The grades in the second stage were weighted more heavily towards the implementation of their solution and evaluation of the results. Table 3 summarizes the degree of completion of the assignment in each stage. In the first stage, only one student (5%) made progress past the initial state with some significant implementation errors. However, in the second stage, nine

students (52%) were able to progress past the initial state, with all but one successfully reaching a reasonable steady state solution. The completed solutions varied somewhat depending on the particular modelling and numerical assumptions used, but all were reasonable. These results show that problem solving skills can indeed be taught, however, there is much room for improvement of the process.

CONCLUSION

Our limited experience with open-ended problems in the three courses discussed here, confirms the results from previous studies [2] that the traditional exercises found in most engineering texts, although useful, do not adequately prepare engineering students for real-world problems. Students seem to have great difficulty approaching these problems, however, they also seem to enjoy the challenge and perform reasonably well if given proper guidance.

Based on our observations, a few open-ended problems sprinkled in each course throughout the curriculum could have a significant impact in:

- Improving students' problem solving skills and in particular their ability to identify and define engineering problems.
- Increasing students' confidence level in approaching real-world problems.
- Making a course more interesting and enjoyable for both the student and the instructor.

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