METR 130: Lecture 6

Air Pollution Meteorology& Gaussian Model

Spring Semester, 2011

(May 10, 2011)

TURBULENCE AVERAGING …

Instantaneous **Averaged**

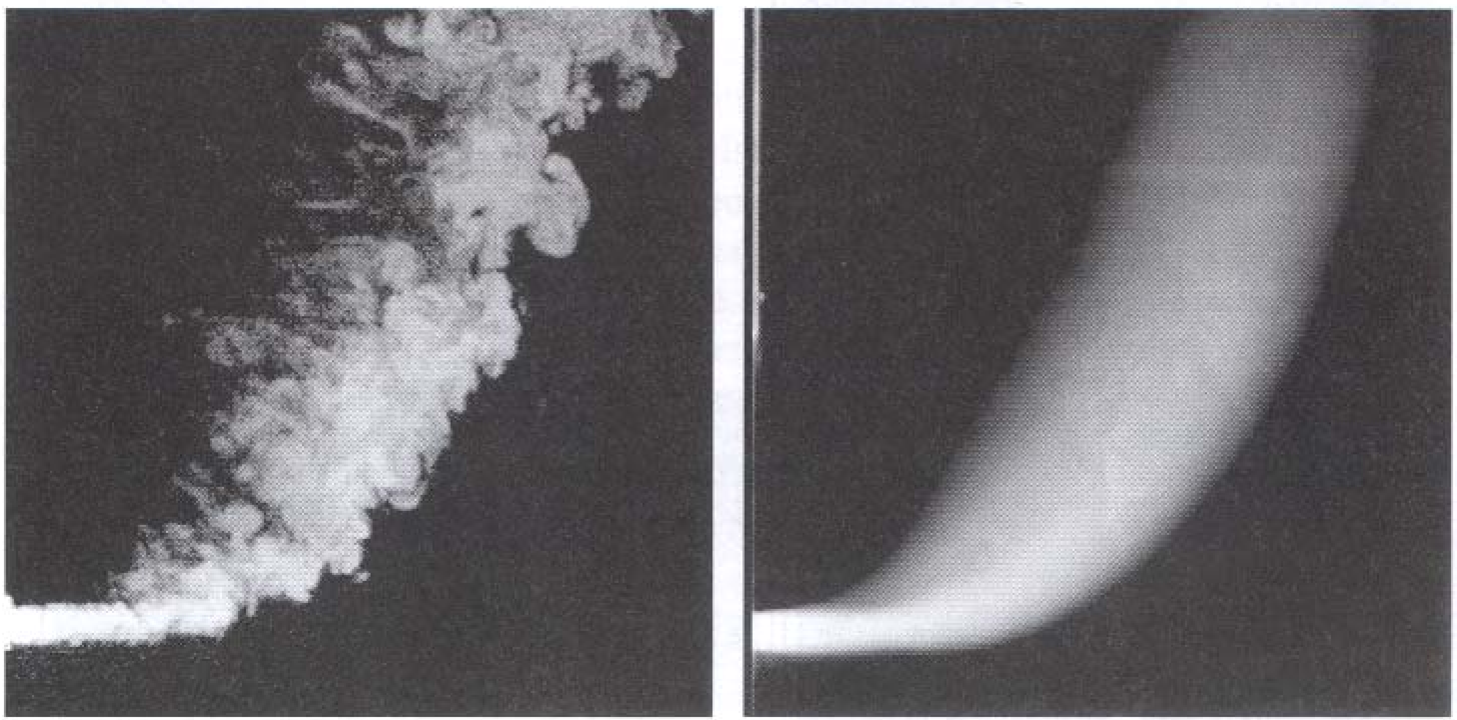
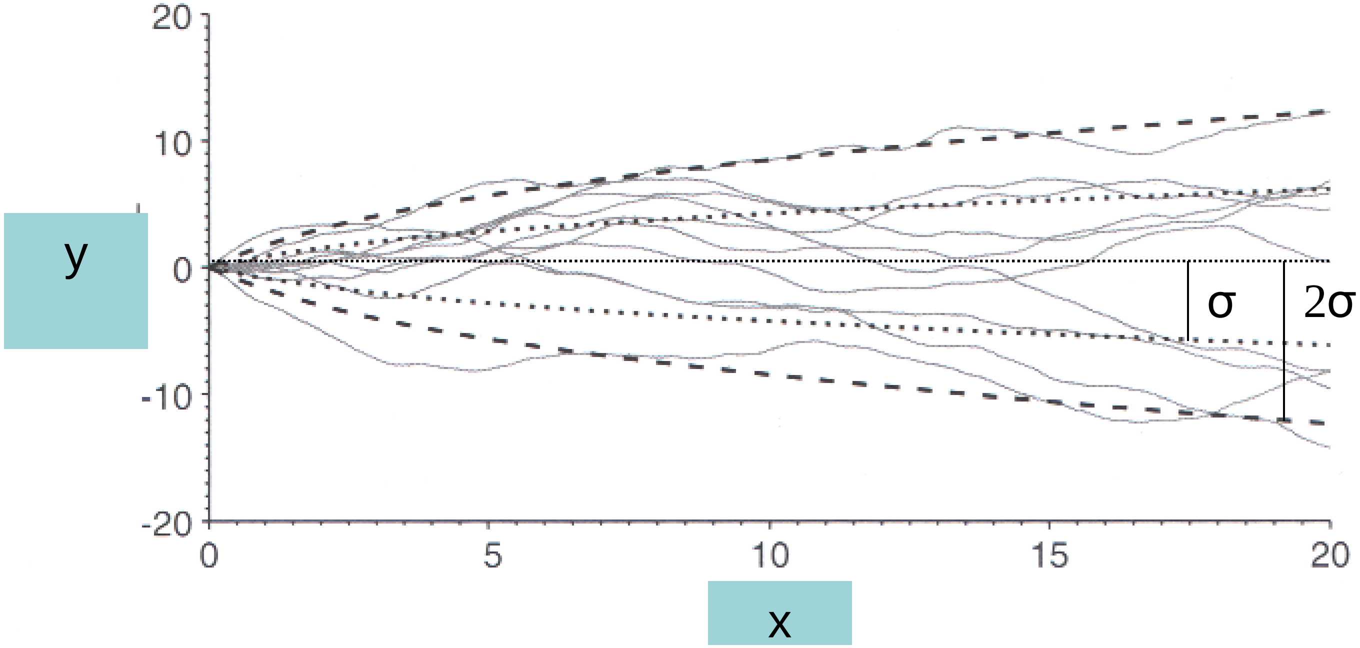


Figure 1.3 *Instantaneous and time averaged views of a jet in cross flow. The jet exits from the wall at left into a stream flowing from bottom to top (Su & Mungal, 1999).*

Spreading of plume as it travels downwind

Each line below is a trajectory of a different particle released from point (x,y) = (0,0) and traveling downwind from left to right across graph. Note spreading with downwind travel of various trajectories. Can characterize the spreading of the plume as the evolution with distance (x) of the standard deviation (σ) of particle positions (y), that is by $\sigma(x)$.

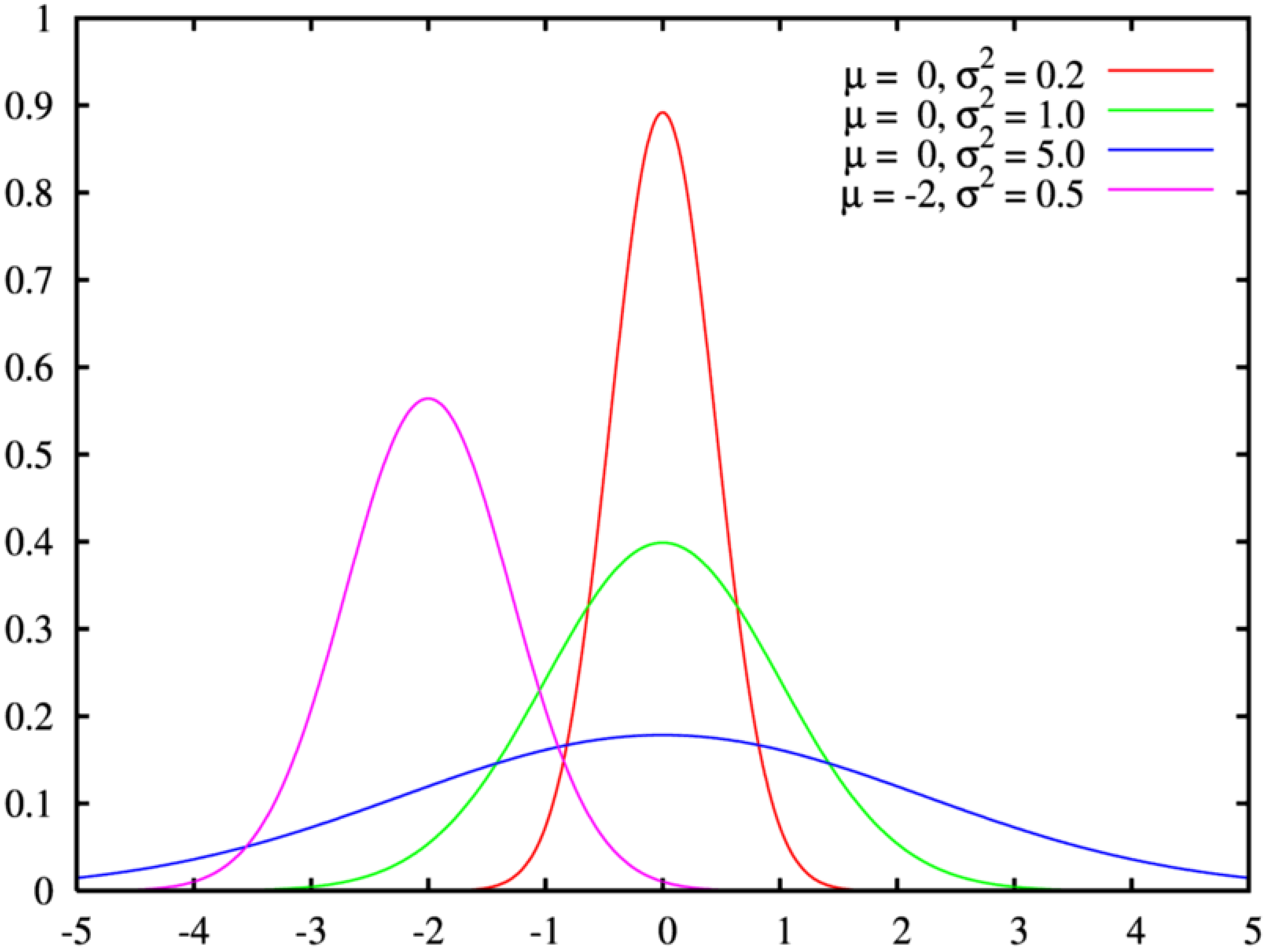


Gaussian Distribution

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

where …

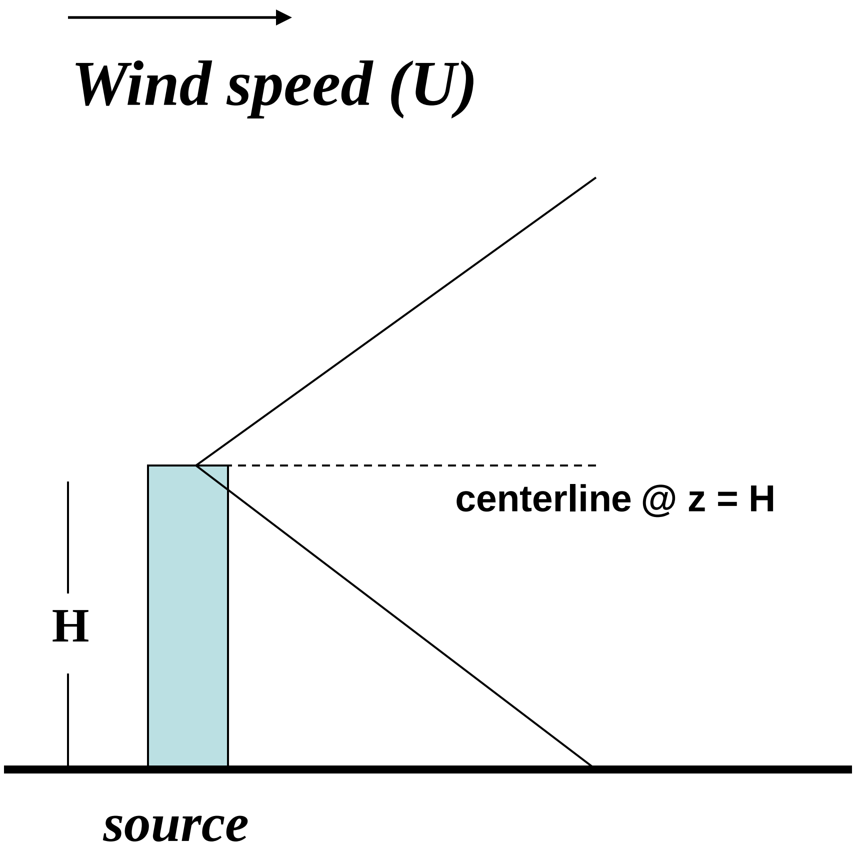
x is an individual sample from a random distribution
P(x) is the probability that an individual sample has a value = x
μ is the mean value of the samples
σ is the standard deviation of the samples



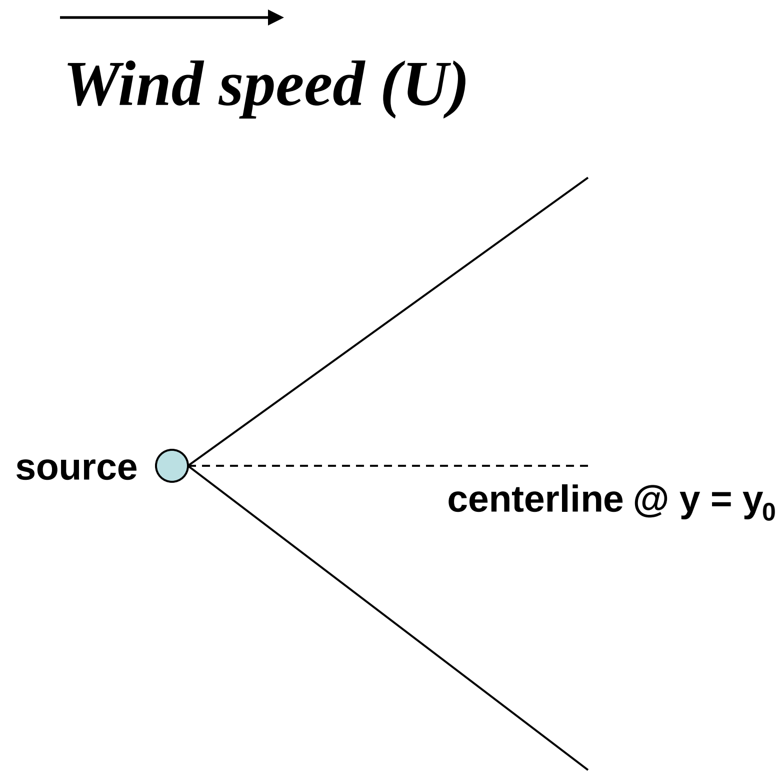
(from Wikipedia: "Google" Normal Distribution)

APPLICATION TO POLLUTION PLUME …

VERTICAL CROSS-SECTION



HORIZONTAL CROSS-SECTION



(note: y_0 often defined as zero)

Gaussian Model: Continuous Source

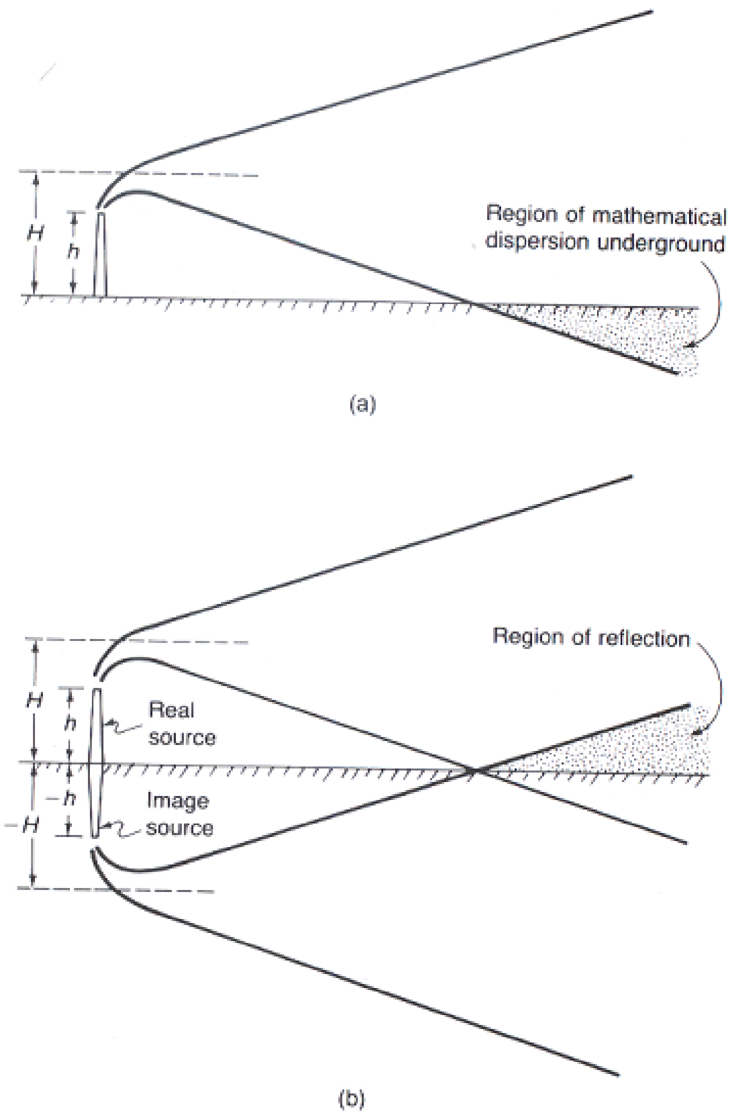
$$C = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left[-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right] \exp\left[-\frac{1}{2}\frac{(z-H)^2}{\sigma_z^2}\right] \qquad (19.2)$$

Gaussian Model: Continuous Source (including surface reflection term)

$$C = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right)\left\{\exp\left(-\frac{1}{2}\frac{(z-H)^2}{\sigma_z^2}\right) + \exp\left(-\frac{1}{2}\frac{(z+H)^2}{\sigma_z^2}\right)\right\}$$

C – Pollutant air concentration (grams per cubic meter)
Q – Emission rate (grams per second)
x – Downwind distance from source (x = 0)
y – Lateral position relative to centerline (y = 0)
z – Height above surface (z = 0)
σ_y – Lateral plume spread (meters)
σ_z – Vertical plume spread (meters)
U – Wind speed (m/s)
H – Effective plume height = $h_s + \Delta h$
h_s – Dource stack height
Δh – Plume rise

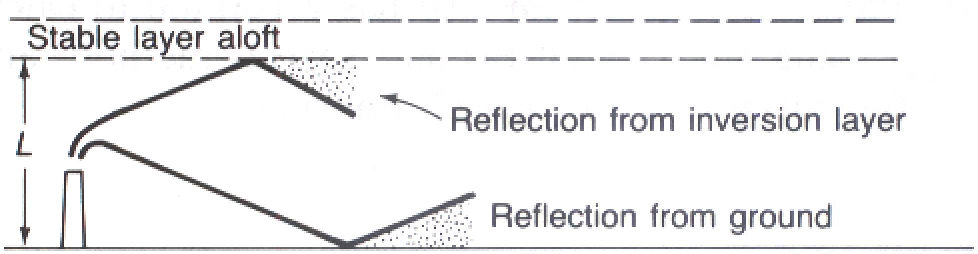
Figure 19.5 Schematic diagram depicting (a) mathematical dispersion of pollutants underground and (b) reflection due to an "image source."



From Cooper & Alley, "Air Pollution Control: A Design Approach"

Gaussian Model: Continuous Source (reflection terms for surface & elevated inversion)

Figure 19.11 Plume dispersion under an elevated inversion.



$$C = \frac{Q}{2\pi u \sigma_y \sigma_z}\left[\exp\left(\frac{-1}{2}\frac{y^2}{\sigma_y^2}\right)\right]\sum_{-\infty}^{+\infty}\left\{\exp\left(\frac{-(z-H+2jL)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z+H+2jL)^2}{2\sigma_z^2}\right)\right\} \quad (19.8)$$

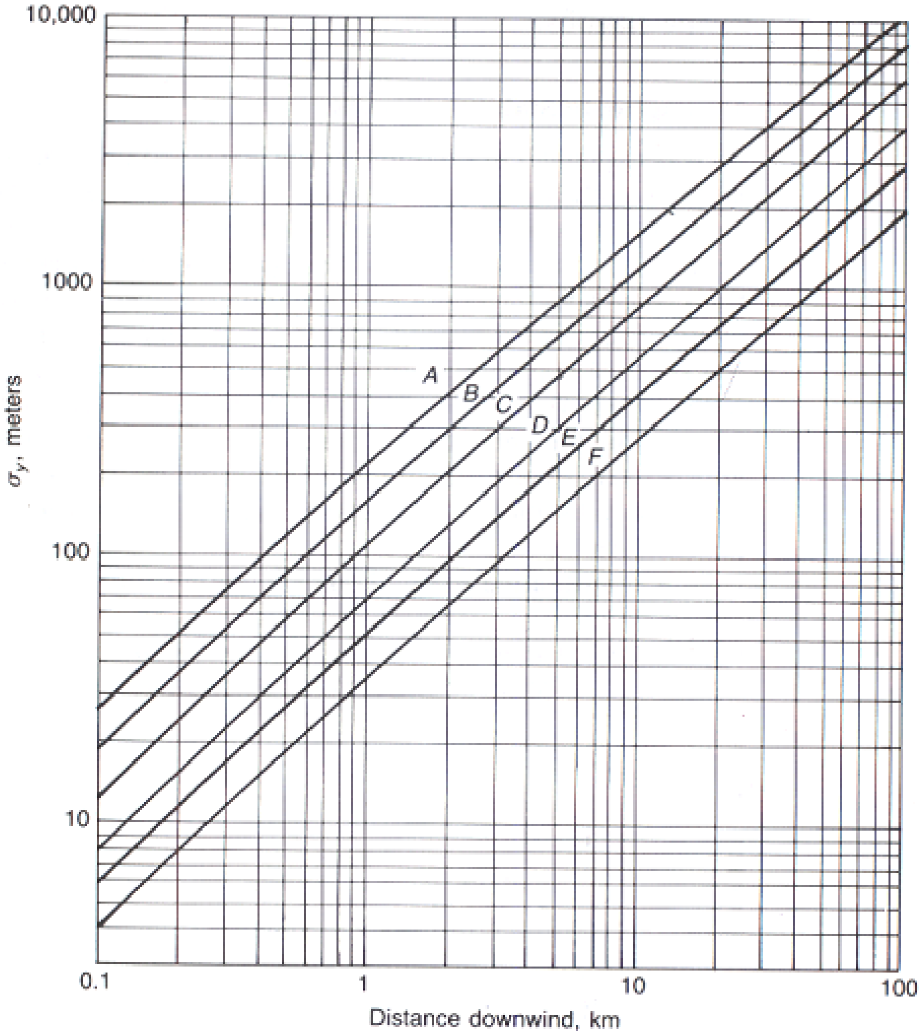
$L = h_i$ = boundary layer depth

Summation is over j, in practice j summed from -2 to 2

From Cooper & Alley, "Air Pollution Control: A Design Approach"

PASQUILL-GIFFORD CURVES: SIGMA-Y

Figure 19.7 Horizontal dispersion coefficient as a function of downwind distance from the source.

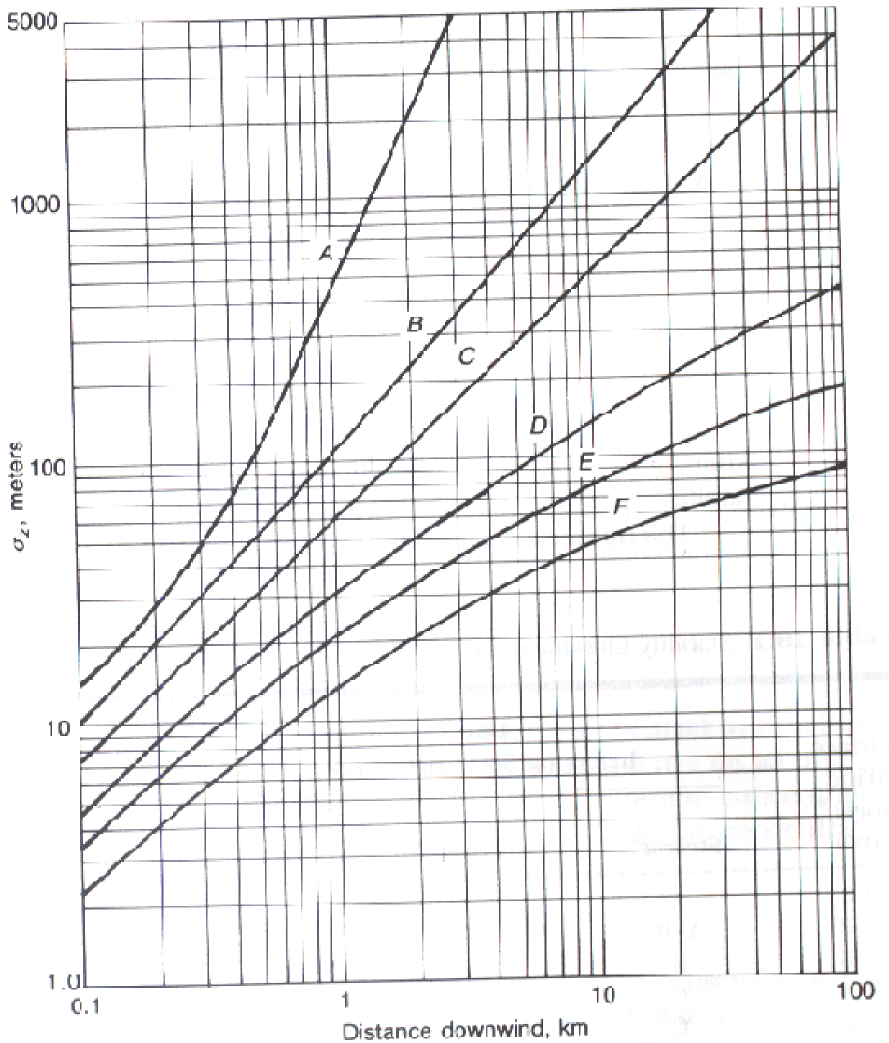


Adapted from Turner, 1970.

From Cooper & Alley, "Air Pollution Control: A Design Approach"

PASQUILL-GIFFORD CURVES: SIGMA-Z

Figure 19.8 Vertical dispersion coefficient as a function of downwind distance from the source.



Adapted from Turner, 1970.

From Cooper & Alley, "Air Pollution Control: A Design Approach"

PASQULL-GIFFORD STABILITY CLASSES

Table 19.1 Stability Classifications*

Surface Wind Speed[a] m/s	Day Incoming Solar Radiation			Night Cloudiness[e]	
	Strong[b]	Moderate[c]	Slight[d]	Cloudy (≥4/8)	Clear (≤3/8)
<2	A	A–B[f]	B	E	F
2–3	A–B	B	C	E	F
3–5	B	B–C	C	D	E
5–6	C	C–D	D	D	D
>6	C	D	D	D	D

NOTES:

a. Surface wind speed is measured at 10 m above the ground.

b. Corresponds to clear summer day with sun higher than 60° above the horizon.

c. Corresponds to a summer day with a few broken clouds, or a clear day with sun 35–60° above the horizon.

d. Corresponds to a fall afternoon, or a cloudy summer day, or clear summer day with the sun 15–35°.

e. Cloudiness is defined as the fraction of sky covered by clouds.

f. For A–B, B–C, or C–D conditions, average the values obtained for each.

* A = Very unstable D = Neutral
B = Moderately unstable E = Slightly stable
C = Slightly unstable F = Stable
Regardless of wind speed, Class D should be assumed for overcast conditions, day or night.

Adapted from Turner, 1970.

From Cooper & Alley, "Air Pollution Control: A Design Approach"

Briggs' Formulas

Table 9.3 Briggs' (1973) Interpolation Formulas for Open Country

Pasquill Type	σ_y (m)	σ_z (m)
A	$0.22x(1 + 0.0001x)^{-1/2}$	$0.20x$*
B	$0.16x(1 + 0.0001x)^{-1/2}$	$0.12x$
C	$0.11x(1 + 0.0001x)^{-1/2}$	$0.08x(1 + 0.0002x)^{-1/2}$
D	$0.08x(1 + 0.0001x)^{-1/2}$	$0.06x(1 + 0.0015x)^{-1/2}$
E	$0.06x(1 + 0.0001x)^{-1/2}$	$0.03x(1 + 0.0003x)^{-1}$
F	$0.04x(1 + 0.0001x)^{-1/2}$	$0.016x(1 + 0.0003x)^{-1}$

*x is the distance downwind of the source in meters.

From Arya, S. P., "Air Pollution Meteorology and Dispersion"

Final Exam (Practice Questions) …

19.1 For an emission rate of 200 g/s, an effective stack height of 80 m, Class C stability, and a wind speed at stack height of 8 m/s, calculate the ground-level concentration of a nonreactive pollutant: (a) 1000 m directly downwind; and (b) 5000 m directly downwind.

Emission Calculation
(for take-home problem)

Emission Rate (Q) = Emission Factor $\times$ Coal Feed Rate $\times$ (1 – Control Efficiency)

kg SO_2 per second   kg SO_2 per Mg Coal   Mg Coal per second

Power Generation = HHV for Coal $\times$ Coal Feed Rate $\times$ Thermal Efficiency

MWatts   MJoules per kg coal   kg coal per second

Various unit conversions are necessary when applying these equations to the take-home problem. Make sure you check footnotes ‘a’ and ‘b’ when looking up AP-42 emission factor in Table 1.1-3.