METR 130: Assignment 3 (Spring 2011)
Due Date: April 21, 2011

Question 1

The "boundary layer" form of the Reynolds-averaged conservation equation for potential temperature is

$$\frac{\partial \bar{\theta}}{\partial t} = -\frac{\partial(\overline{w'\theta'})}{\partial z} + S_\theta^+ - S_\theta^-$$

where S_θ^+ and S_θ^- are source and sink terms, respectively. Derive this equation starting from the instantaneous three-dimensional equation for potential temperature. The procedure is analogous to what was done in lecture notes for the 'u' component of velocity.

Question 2

a. Take two standard dice, the values '1' through '6' of one dice representing the instantaneous 'u' component of velocity and the values '1' through '6' of the other representing the instantaneous 'w' component of velocity. Simultaneously role the dice twenty times and record the values of the dice each time under 'u' and 'w' columns. From these values, calculate

   i. $\bar{u}$ and $\bar{w}$ over the twenty rolls.
   ii. u' and w' for each role
   iii. σ_u^2 and σ_w^2 over the twenty rolls
   iv. $\overline{u'w'}$ over the twenty rolls
   v. The correlation coefficient $R_{uw} = \overline{uw}/\sigma_u \sigma_w$over the twenty rolls

b. Calculate the value of R_{uw} in the near-neutral surface layer given the relationships in equation (10.22) in Arya. Let z = 0 in these formulations to obtain relationships for σ_u and σ_w valid for the surface layer. Let $\overline{u'w'} = -u_*^2$.

c. What are values reported for various correlation coefficients in Section 8.5 and Example Problem 1 in Arya?

d. Based on your work in parts 'a', 'b' and 'c', how many times bigger are correlation coefficients in typical, correlated turbulence in the atmospheric boundary layer compared to truly random turbulence?

Question 3

Determine a typical value for subsidence velocity w_{sub} (cm/sec) that is necessary for a constant ABL depth during typical midday conditions. To do this, assume the following …

- A value for mean potential temperature jump $(\Delta\theta)_e$ determined from the Wangara Day 33 mean potential temperature profiles shown in lecture slides.
- Entrainment flux $(w'\theta')_e = -\beta(w'\theta')_0$, where $\beta = 0.2$ (this is a typically assumed relationship).
- A midday value for surface kinematic heat flux $(w'\theta')_0$ determined from the Wangara Day 33 kinematic heat flux profiles shown in lecture slides.